Post-Quantum Public-Key Pseudorandom Correlation Functions for OT

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Mahshid Riahinia[3]

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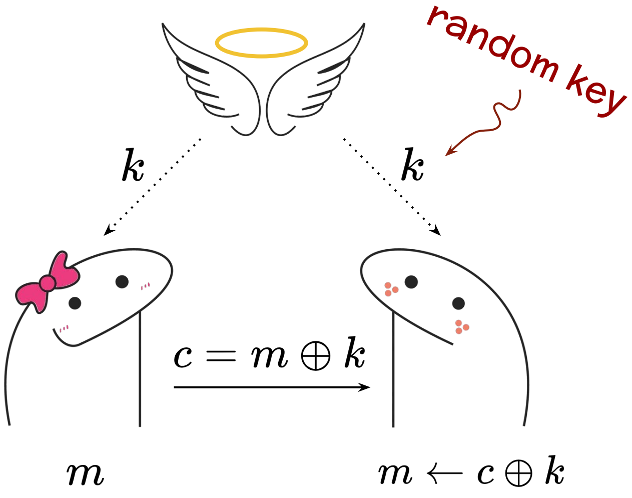
[2] Université Paris Cité, CNRS, IRIF, Paris, France.
[3] ENS, CNRS, visitor at IRIF, Paris, France.

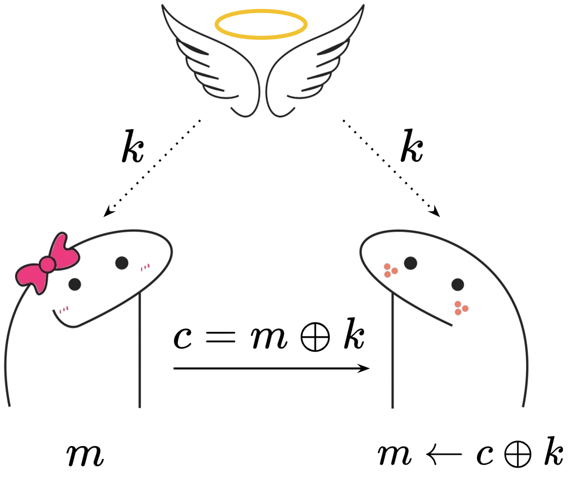
Introduction: Correlated Randomness

Introduction: Correlated Randomness



Symmetric Encryption

Introduction: Correlated Randomness



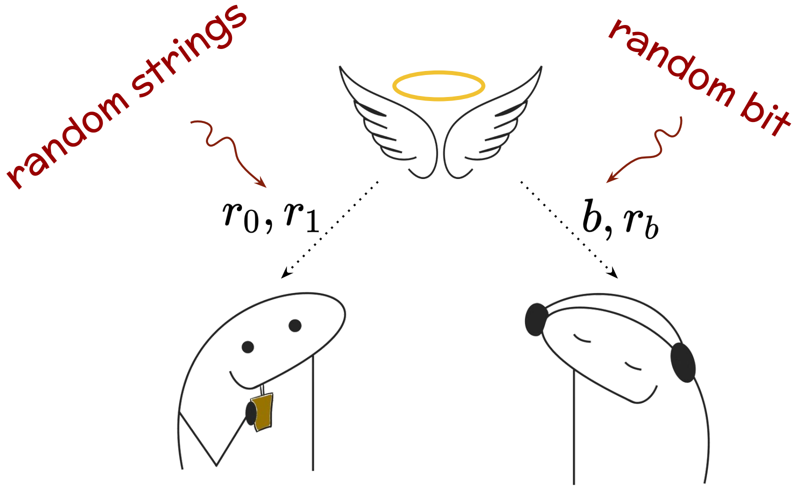
k k

$c = m \oplus k$

m

$m \leftarrow c \oplus k$

Symmetric Encryption



random strings

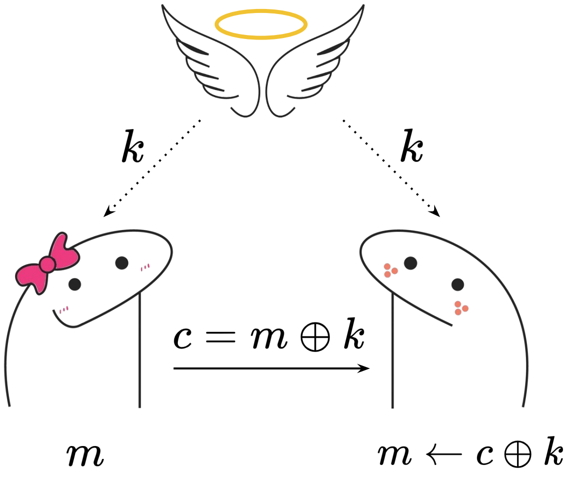
random bit

r_0, r_1

b, r_b

Oblivious Transfer (OT) Correlation

Introduction: Correlated Randomness



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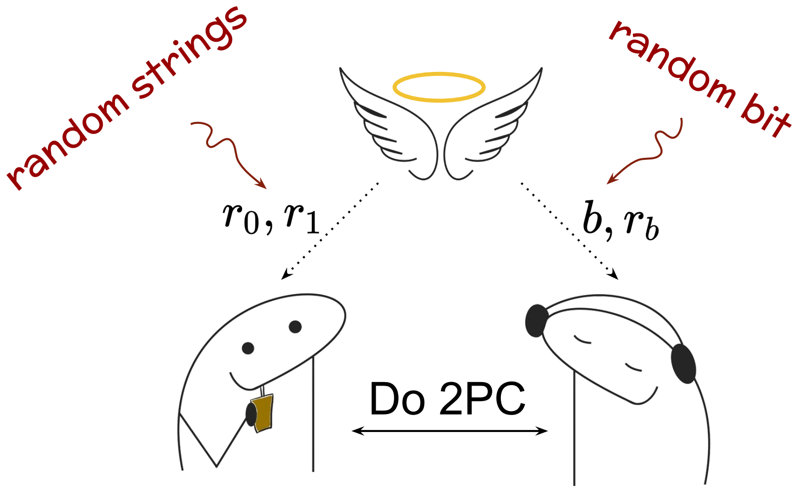
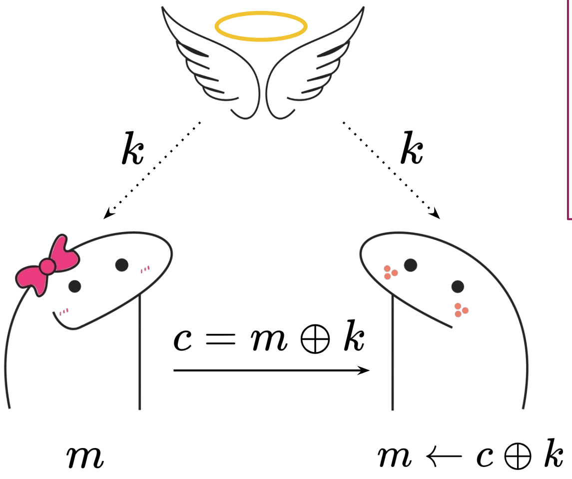


Oblivious Transfer (OT) Correlation

2PC: 2-Party Computation

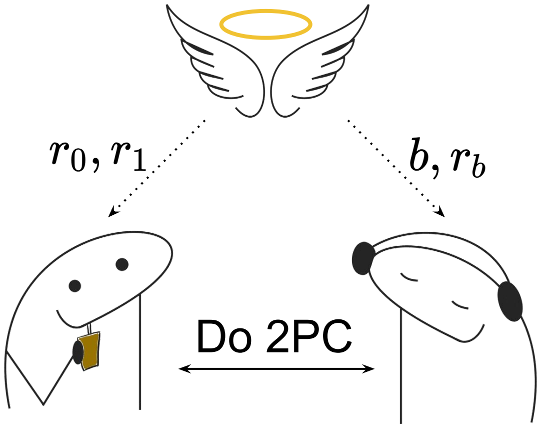
Introduction: Correlated Randomness



Symmetric Encryption

Secure Computation

- party 1 has x
- party 2 has y
- Goal: compute f(x,y) without revealing x,y



Oblivious Transfer (OT) Correlation

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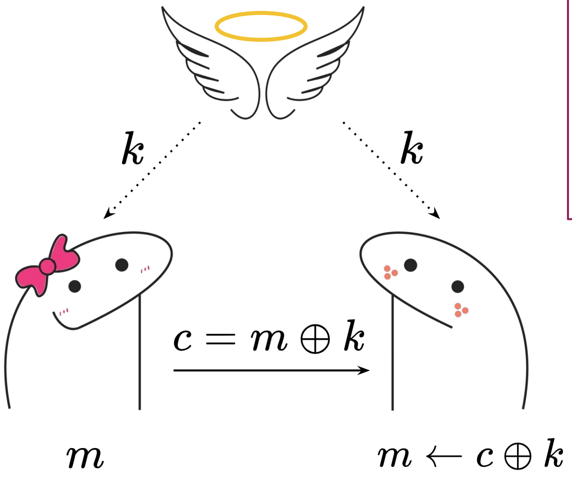
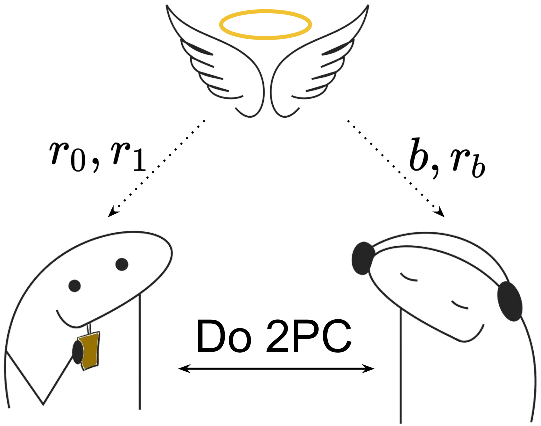


Symmetric Encryption



Oblivious Transfer (OT) Correlation

Secure Communication

Secure Computation

2PC: 2-Party Computation

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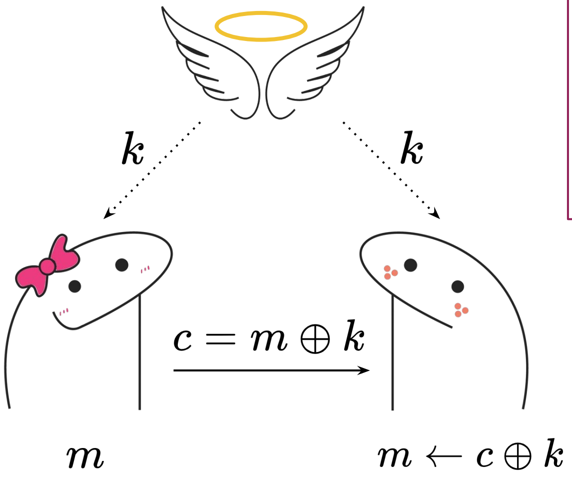

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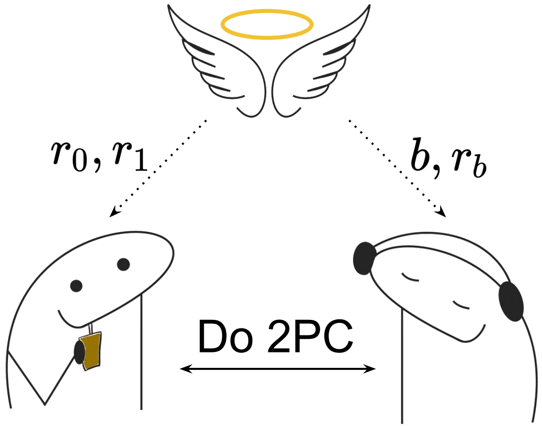


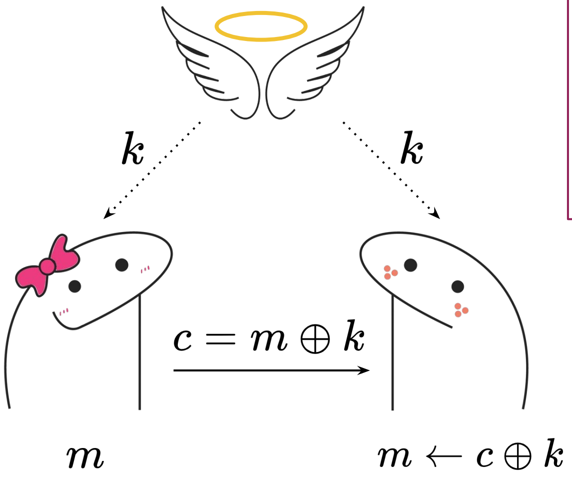
Oblivious Transfer (OT) Correlation

Secure Communication

Fast & Info-Theoretic Secure Computation

2PC: 2-Party Computation

Introduction: Correlated Randomness



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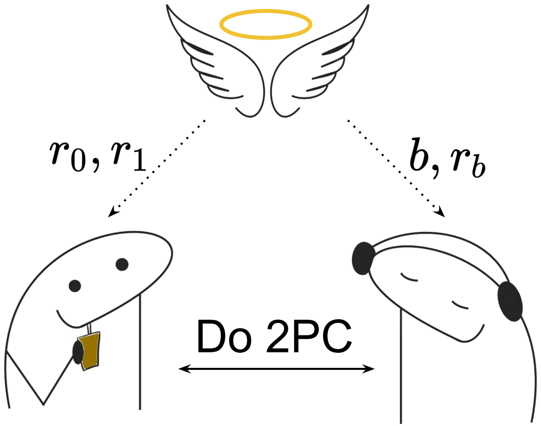


Oblivious Transfer (OT) Correlation

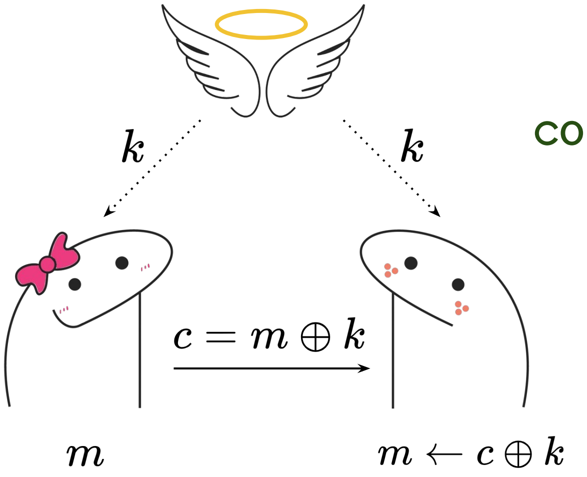
Secure Communication

Fast & Info-Theoretic Secure Computation

2PC: 2-Party Computation

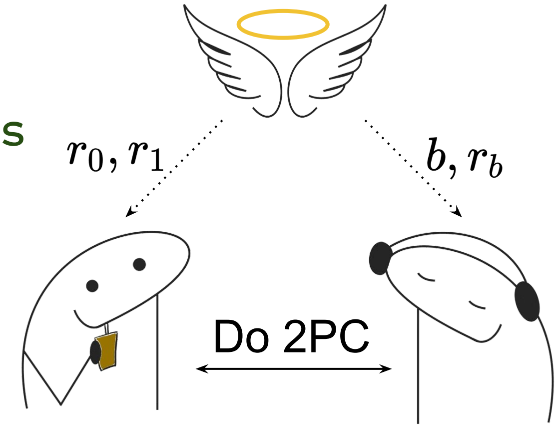
f has n gates : O(n) OT pairs -> send 4 bits per AND

Introduction: Correlated Randomness



Symmetric Encryption

Can we generate correlated randomness on demand?



Oblivious Transfer (OT) Correlation

Introduction: Correlated Randomness

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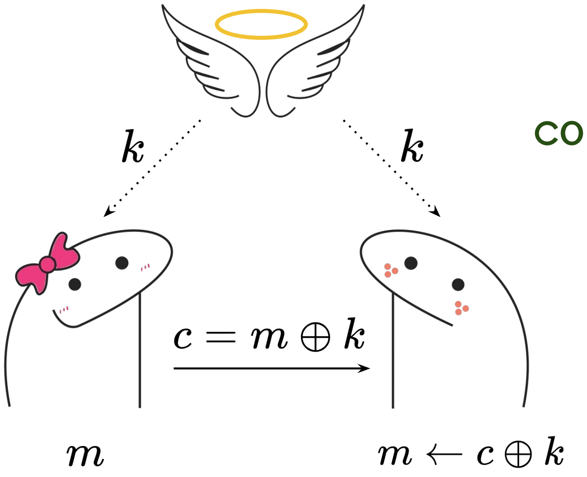
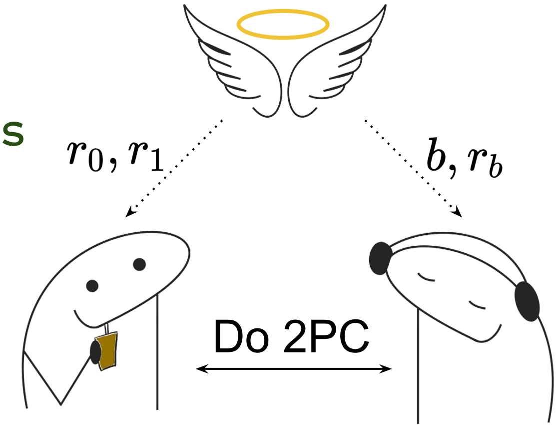


Symmetric Encryption



Oblivious Transfer (OT) Correlation

[BCGIKS 19] Pseudorandom Correlation Generators (PCGs)
[BCGIKS 20] Pseudorandom Correlation Functions (PCFs)

Introduction: Correlated Randomness

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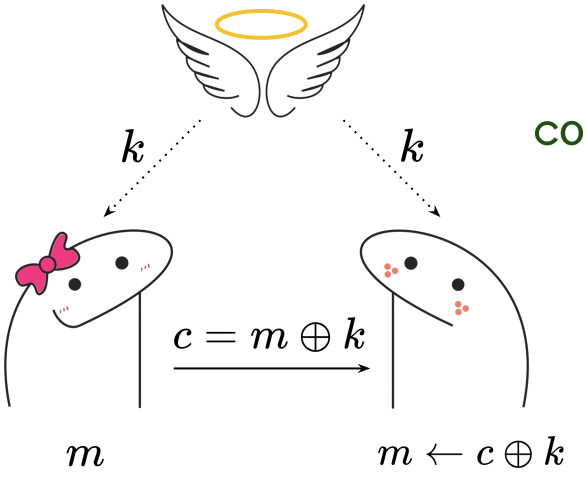
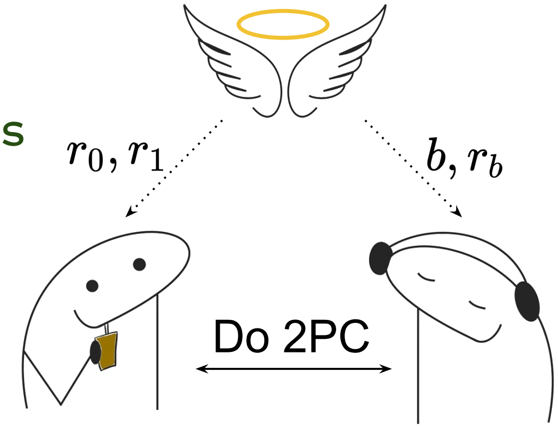


Symmetric Encryption



Oblivious Transfer (OT) Correlation

[BCGIKS 19] Pseudorandom Correlation Generators (PCGs)

➡ [BCGIKS 20] Pseudorandom Correlation Functions (PCFs)

Pseudorandom Correlation Functions

Definition

Pseudorandom Correlation Functions [BCGIKS20]

on-demand generation
of
correlated randomness

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on-demand generation
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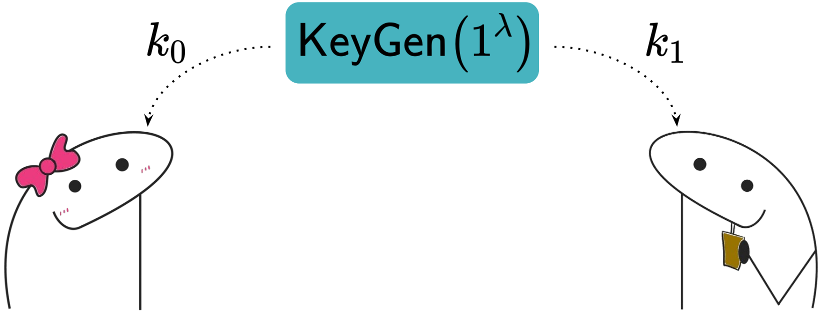


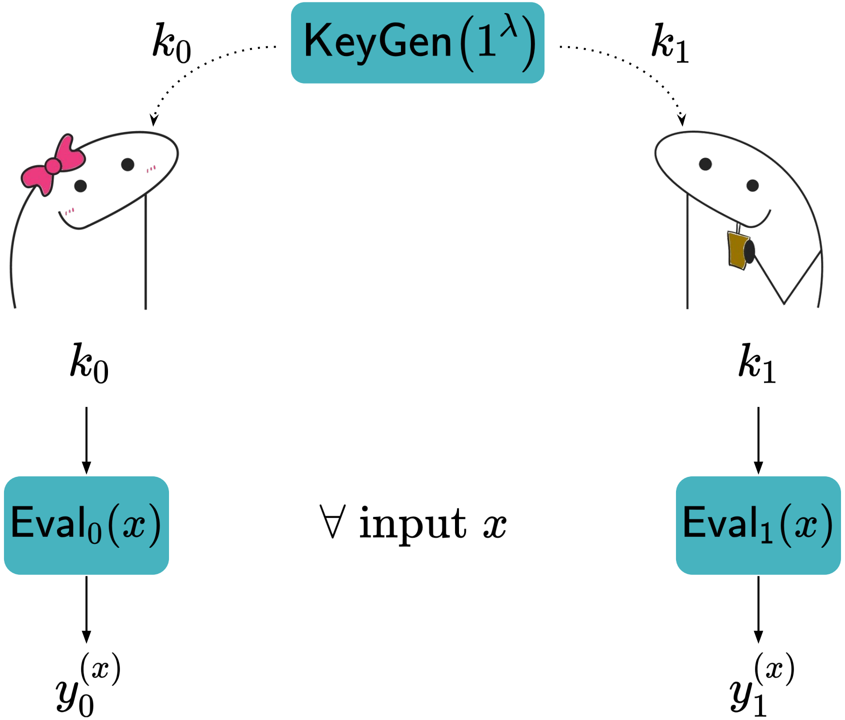
$\mathsf{KeyGen}(1^\lambda)$

k_0

k_1

Pseudorandom Correlation Functions [BCGIKS20]

on-demand generation
of
correlated randomness



$\mathsf{KeyGen}(1^\lambda)$

k_0 k_1

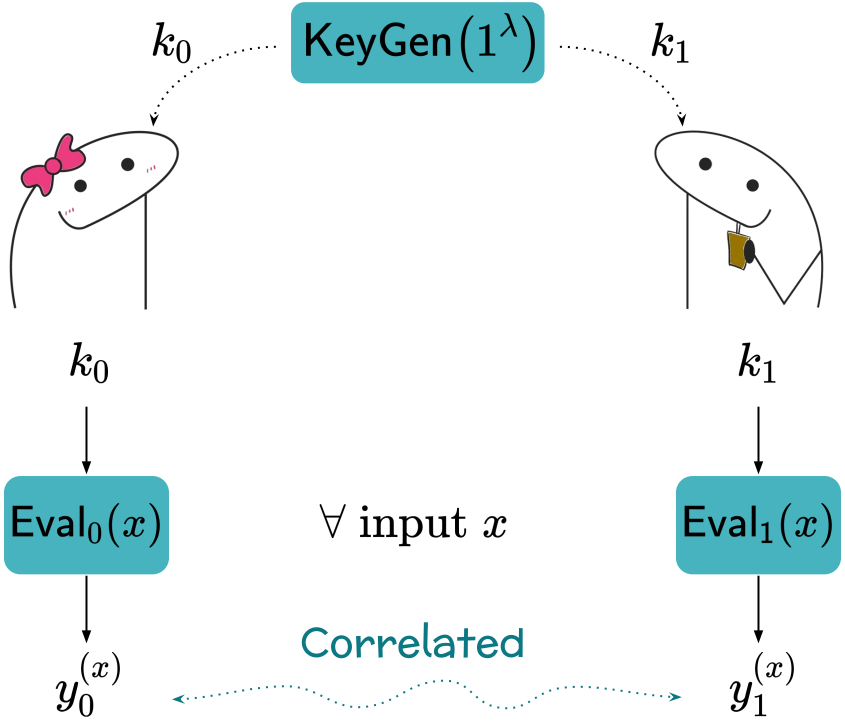
k_0 k_1

$\mathsf{Eval}_0(x)$ $\forall$ input x $\mathsf{Eval}_1(x)$

$y_0^{(x)}$ $y_1^{(x)}$

Pseudorandom Correlation Functions [BCGIKS20]

on-demand generation
of
correlated randomness



KeyGen(1^λ)

k_0 k_1

k_0 k_1

Eval$_0(x)$ $\forall$ input x Eval$_1(x)$

Correlated

$y_0^{(x)}$ $y_1^{(x)}$

Pseudorandom Correlation Functions [BCGIKS20]

on-demand generation
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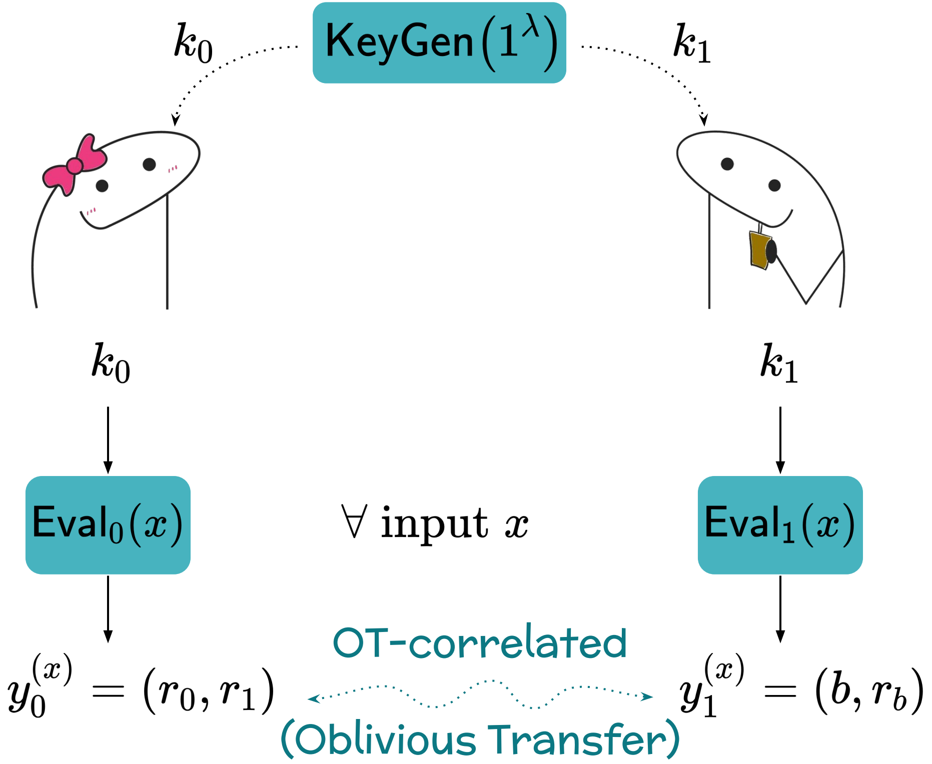
correlated randomness



KeyGen(1^λ)

k_0

k_1

k_0

k_1

Eval$_0(x)$

$\forall$ input x

Eval$_1(x)$

$y_0^{(x)} = (r_0, r_1)$

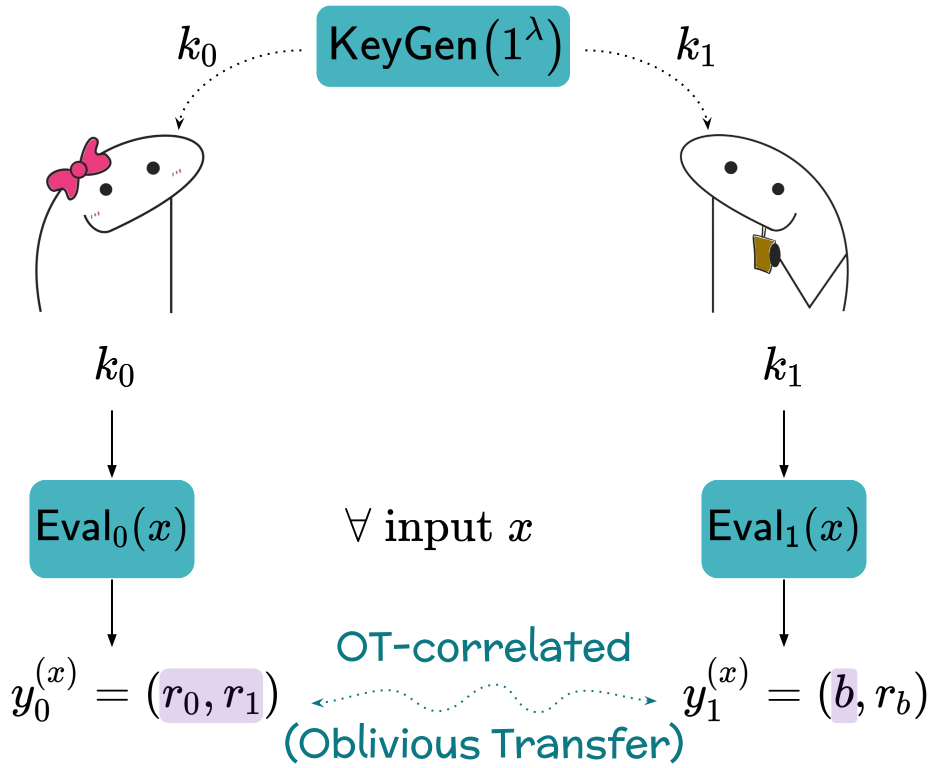
OT-correlated

(Oblivious Transfer)

$y_1^{(x)} = (b, r_b)$

Pseudorandom Correlation Functions [BCGIKS20]

on-demand generation
of
correlated randomness



k_0 KeyGen(1^λ) k_1

k_0 k_1

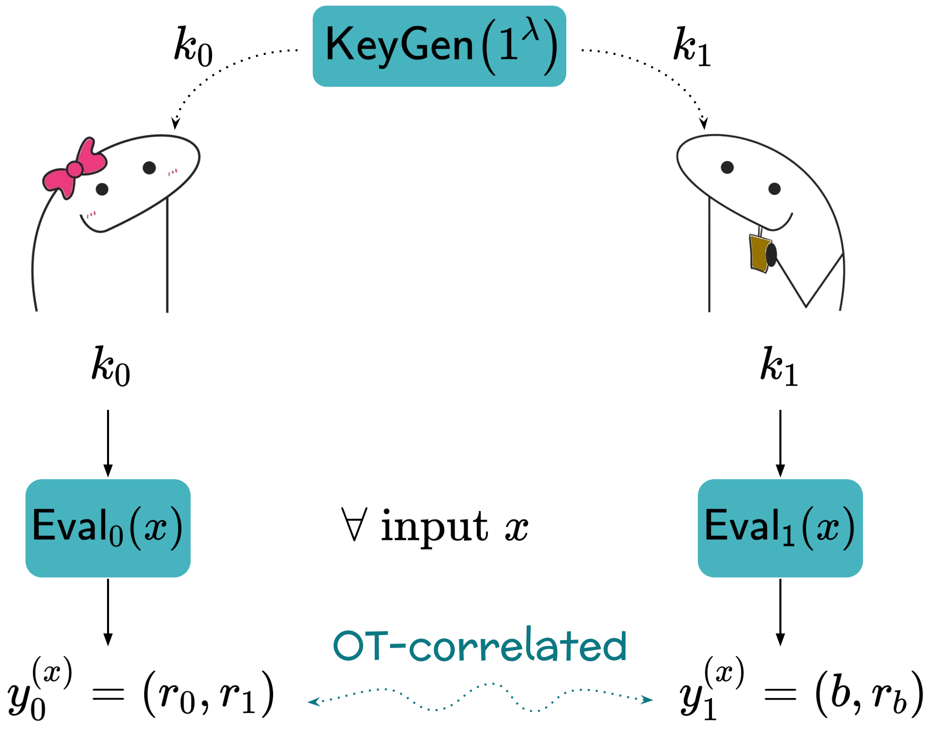
Eval$_0(x)$ $\forall$ input x Eval$_1(x)$

random strings $y_0^{(x)} = (r_0, r_1)$ OT-correlated (Oblivious Transfer) $y_1^{(x)} = (b, r_b)$ random bit

Pseudorandom Correlation Functions [BCGIKS20]

security:

things look random up to correlation



$\mathsf{KeyGen}(1^\lambda)$

k_0 k_1

k_0 k_1

$\mathsf{Eval}_0(x)$ $\forall$ input x $\mathsf{Eval}_1(x)$

OT-correlated

$y_0^{(x)} = (r_0, r_1)$ $y_1^{(x)} = (b, r_b)$

Pseudorandom Correlation Functions [BCGIKS20]

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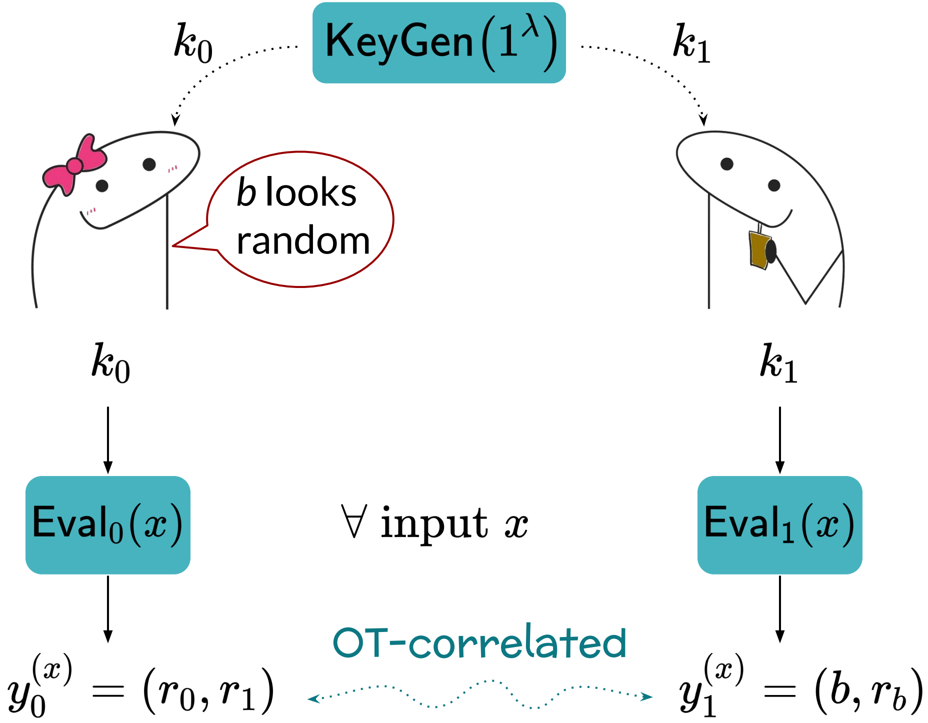

KeyGen(1^λ)

k_0

k_1

b looks random

k_0

k_1

Eval$_0(x)$

$\forall$ input x

Eval$_1(x)$

OT-correlated

$y_0^{(x)} = (r_0, r_1)$

$y_1^{(x)} = (b, r_b)$

Pseudorandom Correlation Functions [BCGIKS20]

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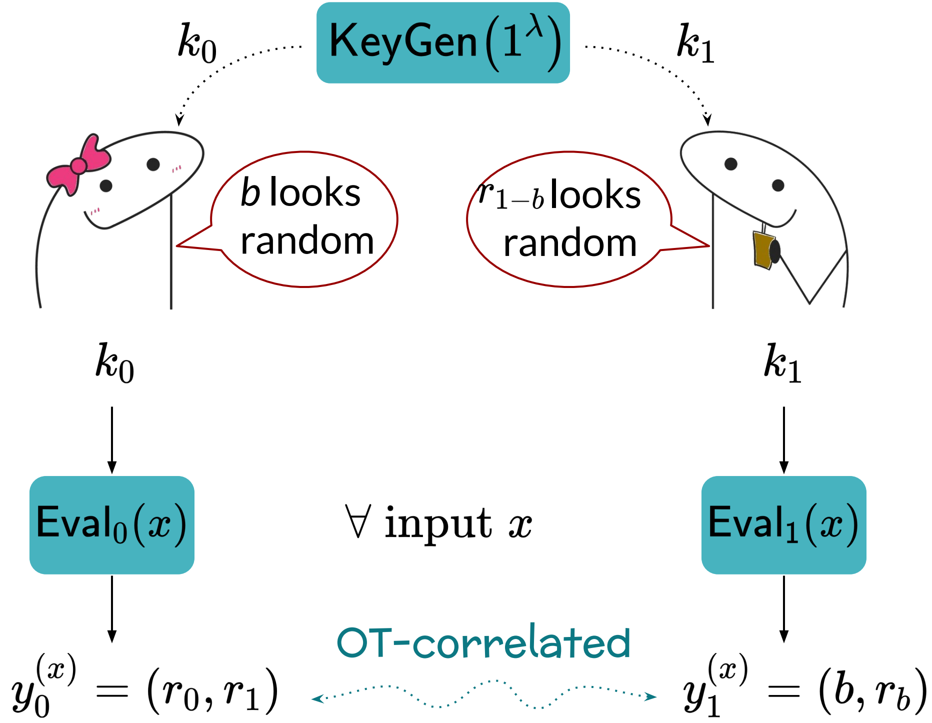


KeyGen(1^λ)

k_0

k_1

b looks random

r_{1-b} looks random

k_0

k_1

Eval$_0(x)$

$\forall$ input x

Eval$_1(x)$

$y_0^{(x)} = (r_0, r_1)$

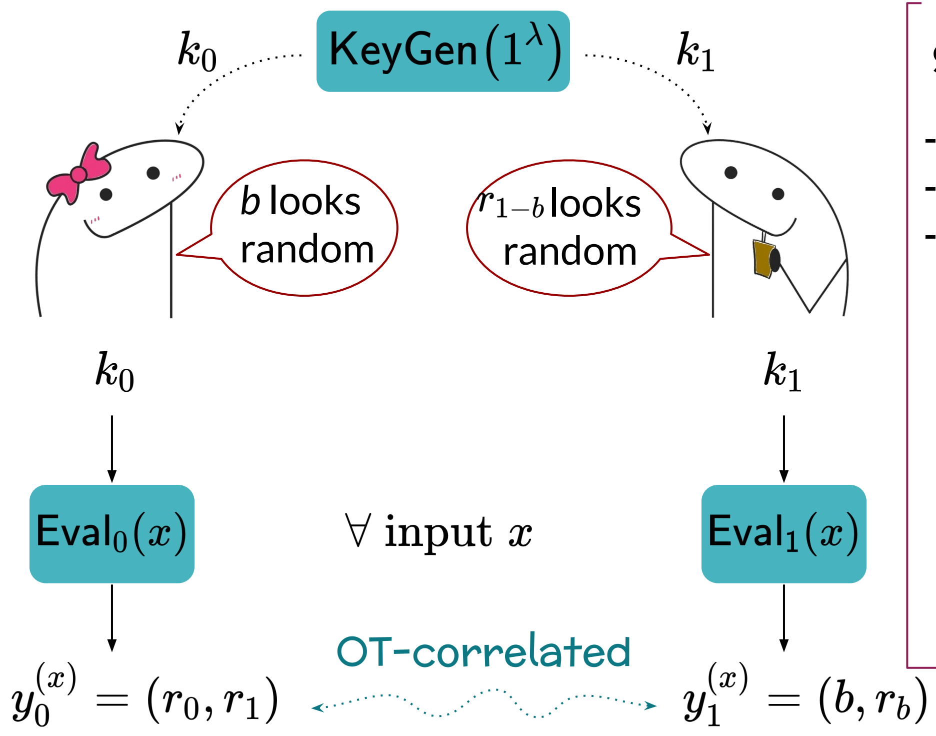
OT-correlated

$y_1^{(x)} = (b, r_b)$

Pseudorandom Correlation Functions [BCGIKS20]

security:

things look random up to correlation



k_0 KeyGen(1^λ) k_1

b looks random

r_{1-b} looks random

k_0 k_1

Eval$_0(x)$ $\forall$ input x Eval$_1(x)$

OT-correlated

$y_0^{(x)} = (r_0, r_1)$ $y_1^{(x)} = (b, r_b)$

Application

Secure Computation

- party 1 has x
- party 2 has y
- Goal: compute f(x,y) without revealing x,y

[GMW87]

|f|=n :

O(n) OT correlations

⇓

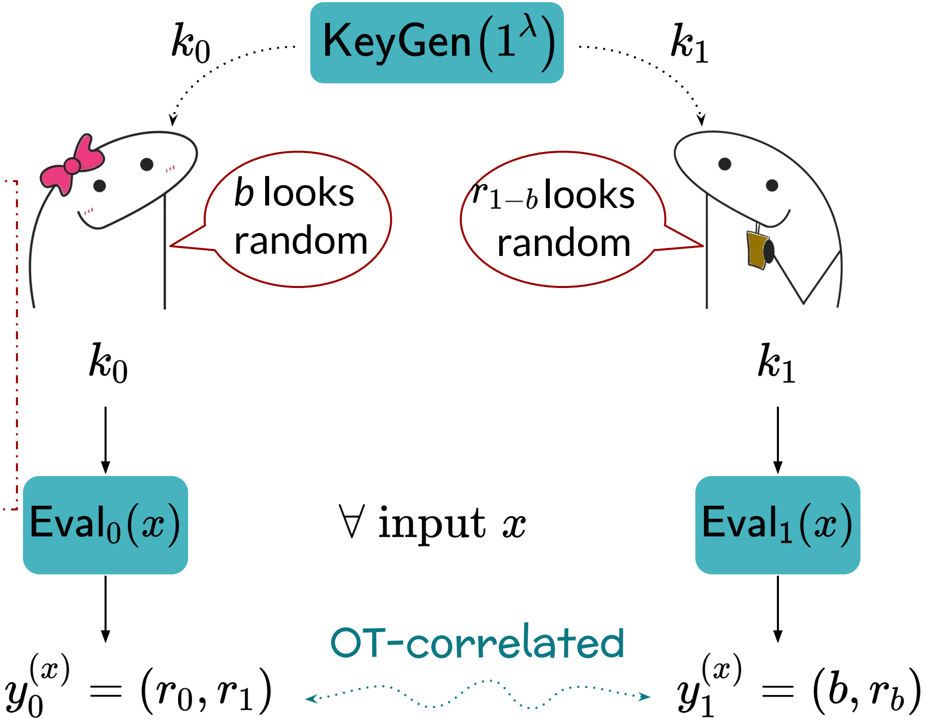
send 4 bits/AND

Pseudorandom Correlation Functions [BCGIKS20]

Our contribution:

Efficient* Post-Quantum Public-Key PCF
for
OT Correlations

* Efficiency ~ #OT per second



$\mathsf{KeyGen}(1^\lambda)$

k_0 k_1

k_0 → $\mathsf{Eval}_0(x)$

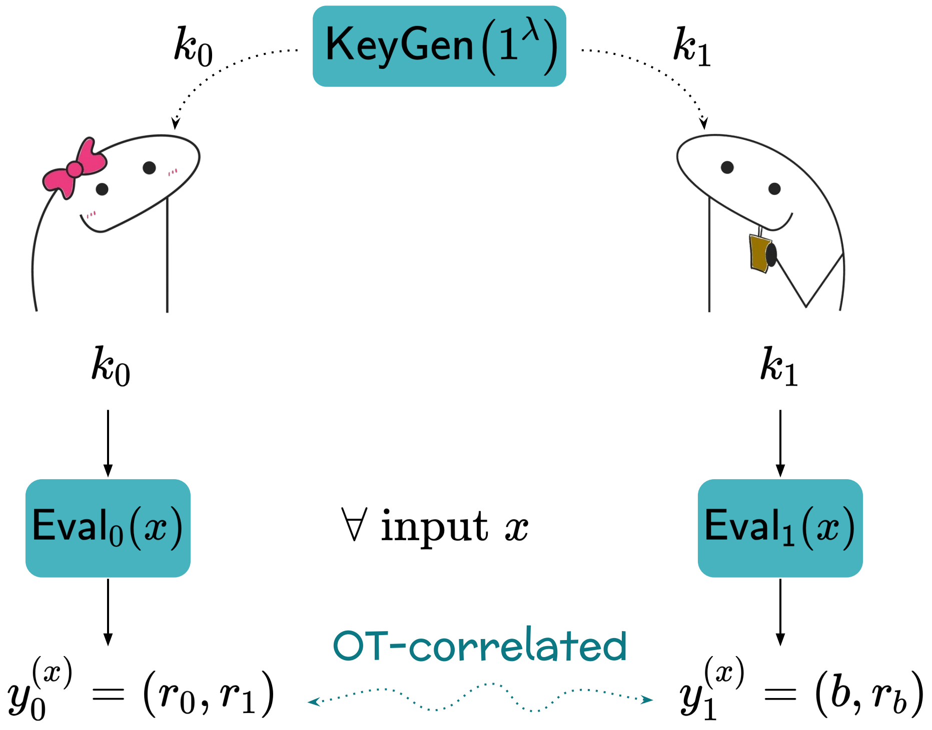
k_1 → $\mathsf{Eval}_1(x)$

$\forall$ input x

$y_0^{(x)} = (r_0, r_1)$ ⟷ OT-correlated ⟷ $y_1^{(x)} = (b, r_b)$

Public-Key Pseudorandom Correlation Functions

Pseudorandom Correlation Functions [BCGIKS20]



$\mathsf{KeyGen}(1^\lambda)$

k_0 k_1

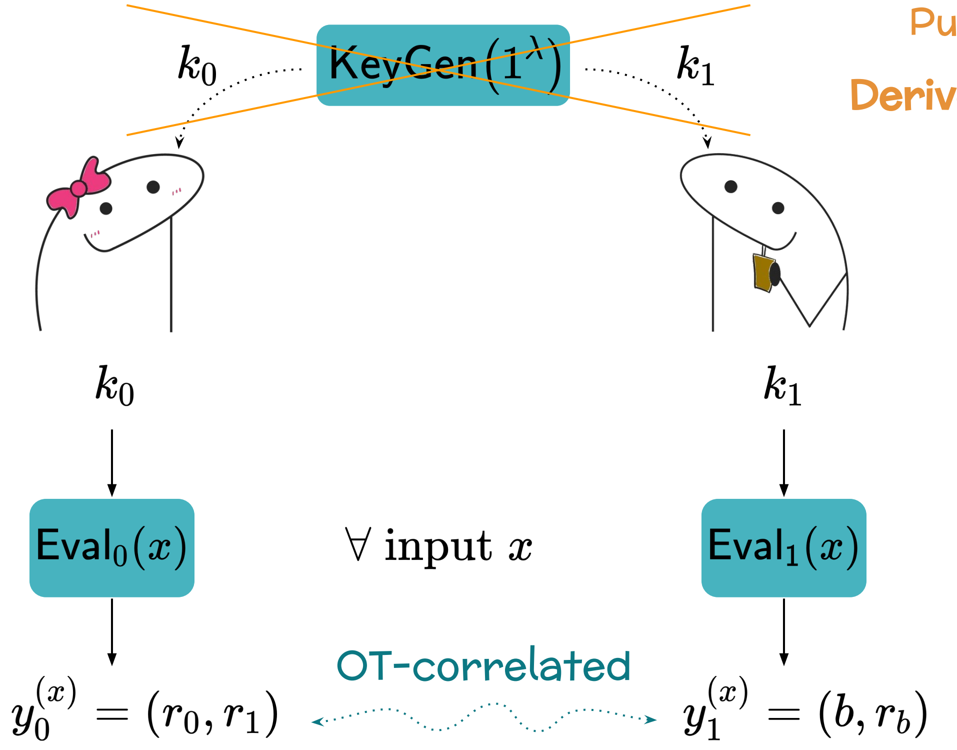
k_0 k_1

$\mathsf{Eval}_0(x)$ $\forall$ input x $\mathsf{Eval}_1(x)$

OT-correlated

$y_0^{(x)} = (r_0, r_1)$ $y_1^{(x)} = (b, r_b)$

Public-Key Pseudorandom Correlation Functions [BCMPR24]



~~$\mathsf{KeyGen}(1^\lambda)$~~

k_0 k_1

k_0 k_1

$\mathsf{Eval}_0(x)$ $\forall$ input x $\mathsf{Eval}_1(x)$

OT-correlated

$y_0^{(x)} = (r_0, r_1)$ $y_1^{(x)} = (b, r_b)$

Public-Key PCF:

Derive keys publicly

Public-Key Pseudorandom Correlation Functions [BCMPR24]

Public-Key Pseudorandom Correlation Functions [BCMP**R**24]

Key Derivation

Evaluation

Public-Key Pseudorandom Correlation Functions [BCMPR24]

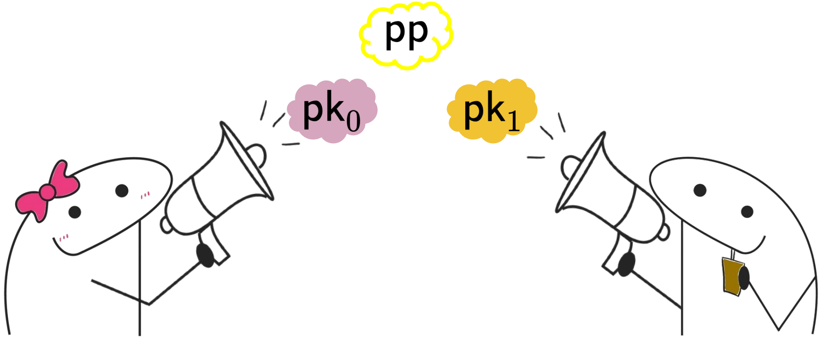
pp

Key Derivation

Evaluation

Public-Key Pseudorandom Correlation Functions [BCMPR24]

Key Derivation



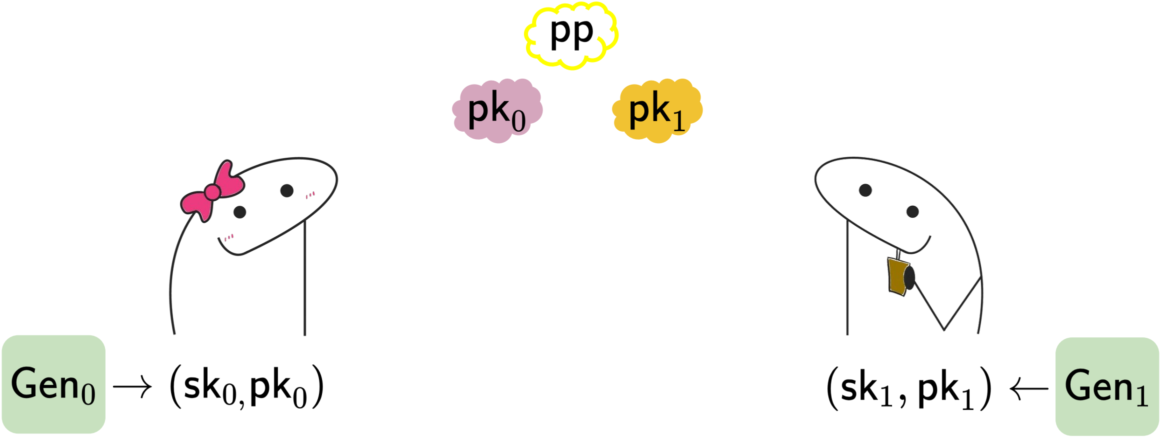
$\mathsf{Gen}_0 \rightarrow (\mathsf{sk}_0, \mathsf{pk}_0)$

$(\mathsf{sk}_1, \mathsf{pk}_1) \leftarrow \mathsf{Gen}_1$

Evaluation

Public-Key Pseudorandom Correlation Functions [BCMPR24]

Key Derivation



pp

pk_0 pk_1

$\boxed{\mathsf{Gen}_0} \rightarrow (\mathsf{sk}_0, \mathsf{pk}_0)$

$(\mathsf{sk}_1, \mathsf{pk}_1) \leftarrow \boxed{\mathsf{Gen}_1}$

Evaluation

Public-Key Pseudorandom Correlation Functions [BCMPR24]

Key Derivation

pp

pk_0 pk_1

$\mathsf{Gen}_0 \rightarrow (\mathsf{sk}_0, \mathsf{pk}_0)$

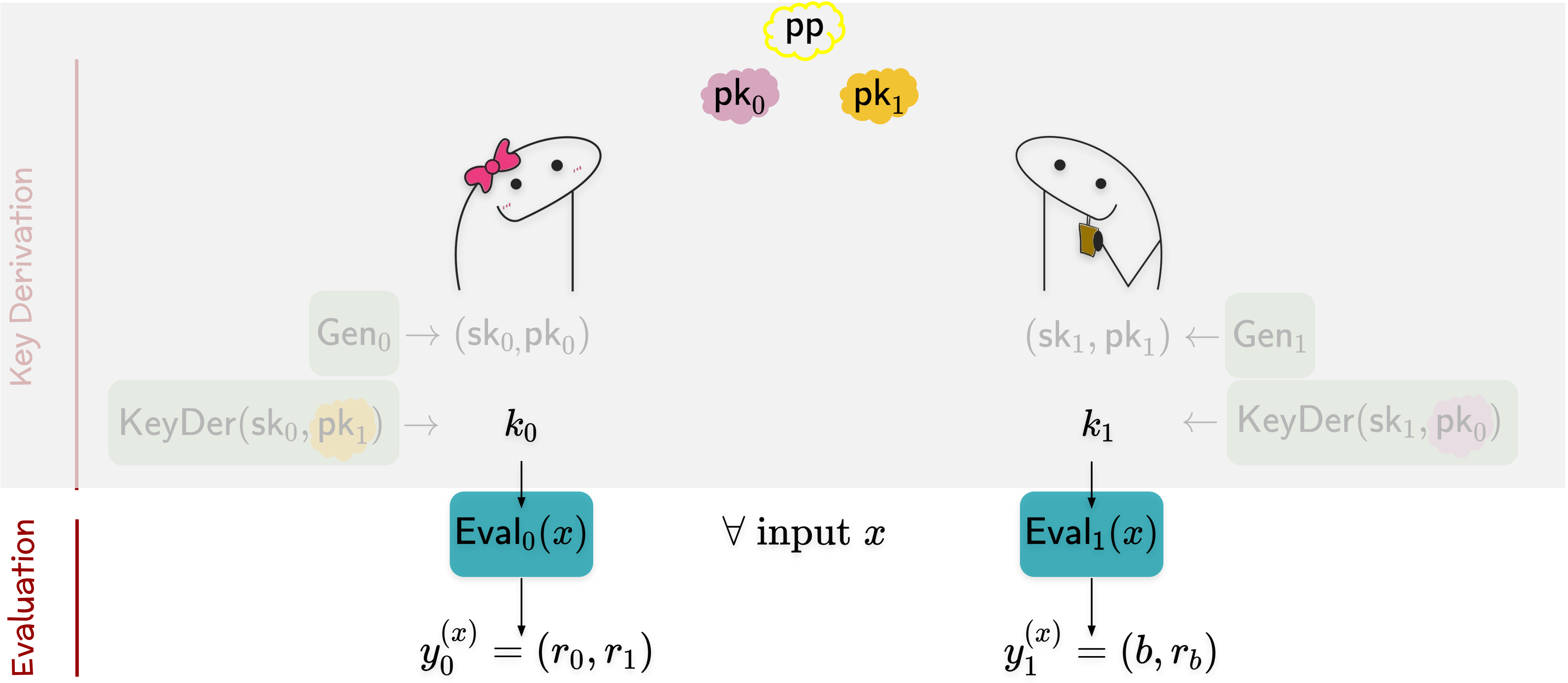
$\mathsf{KeyDer}(\mathsf{sk}_0, \mathsf{pk}_1) \rightarrow k_0$

$(\mathsf{sk}_1, \mathsf{pk}_1) \leftarrow \mathsf{Gen}_1$

$k_1 \leftarrow \mathsf{KeyDer}(\mathsf{sk}_1, \mathsf{pk}_0)$

Evaluation

Public-Key Pseudorandom Correlation Functions [BCMPR24]



pp

pk_0 pk_1

Key Derivation

$\mathsf{Gen}_0 \rightarrow (\mathsf{sk}_0, \mathsf{pk}_0)$

$(\mathsf{sk}_1, \mathsf{pk}_1) \leftarrow \mathsf{Gen}_1$

$\mathsf{KeyDer}(\mathsf{sk}_0, \mathsf{pk}_1) \rightarrow k_0$

$k_1 \leftarrow \mathsf{KeyDer}(\mathsf{sk}_1, \mathsf{pk}_0)$

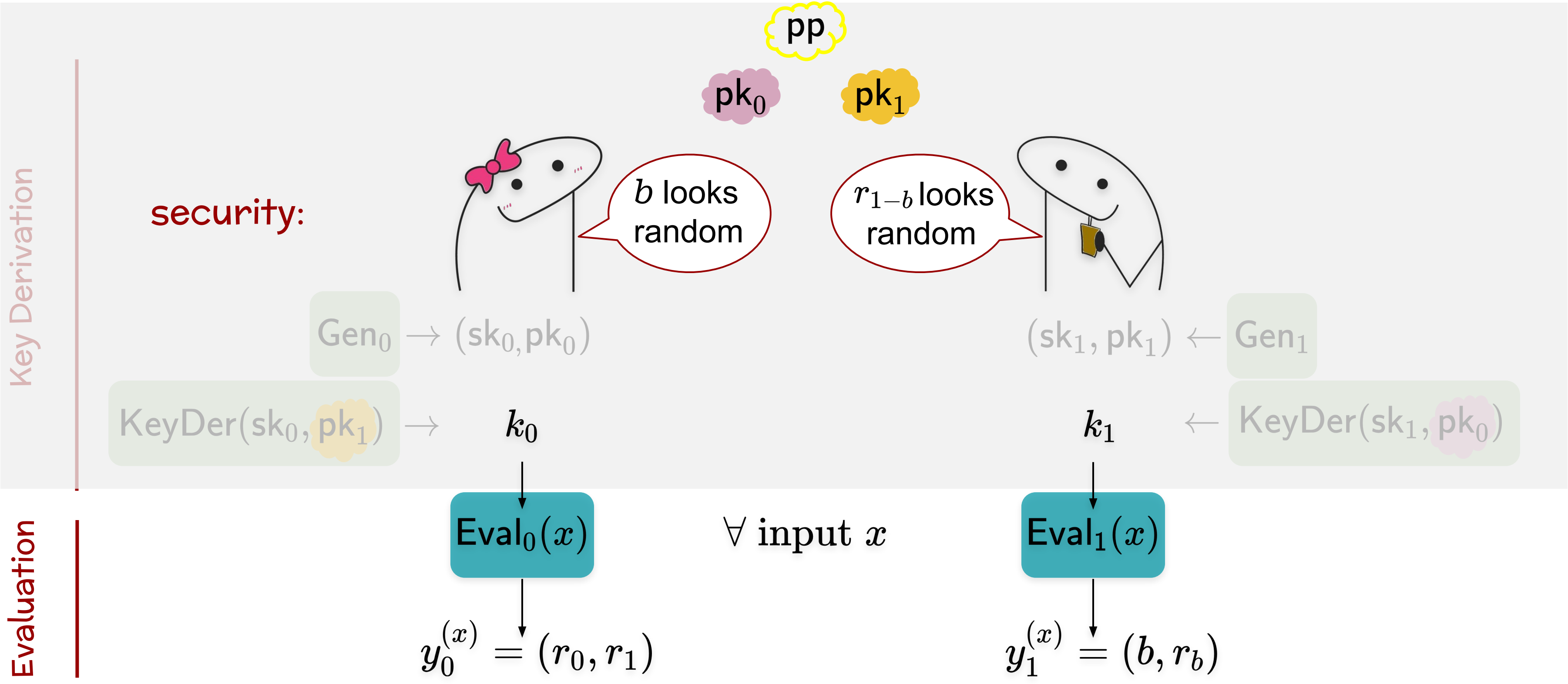
Evaluation

$\mathsf{Eval}_0(x)$ $\forall$ input x $\mathsf{Eval}_1(x)$

$y_0^{(x)} = (r_0, r_1)$

$y_1^{(x)} = (b, r_b)$

Public-Key Pseudorandom Correlation Functions [BCMPR24]



Our Contributions

Contributions

Efficient Public-Key PCF for OT Correlations
from Lattices

Secret-Power Ring
Learning with Errors

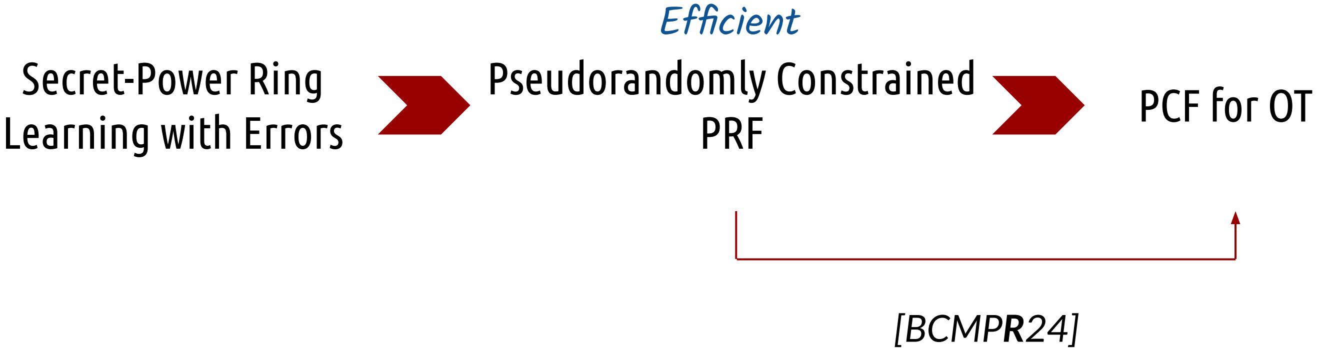
Contributions

Efficient Public-Key PCF for OT Correlations from Lattices

Secret-Power Ring Learning with Errors ➤ *Efficient* Pseudorandomly Constrained PRF

Contributions

Efficient Public-Key PCF for OT Correlations from Lattices



Secret-Power Ring Learning with Errors

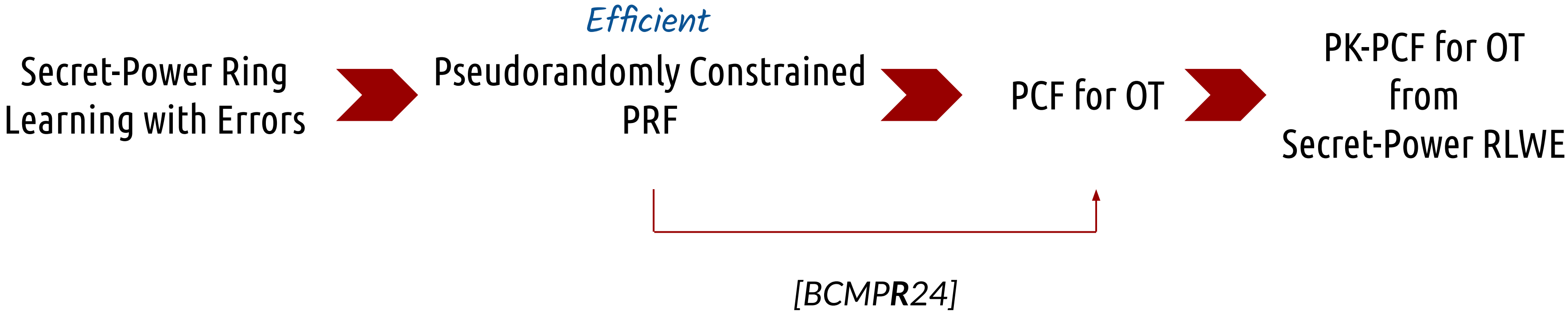
Efficient
Pseudorandomly Constrained PRF

PCF for OT

*[BCMP**R**24]*

Contributions

Efficient Public-Key PCF for OT Correlations from Lattices



Secret-Power Ring Learning with Errors

Efficient Pseudorandomly Constrained PRF

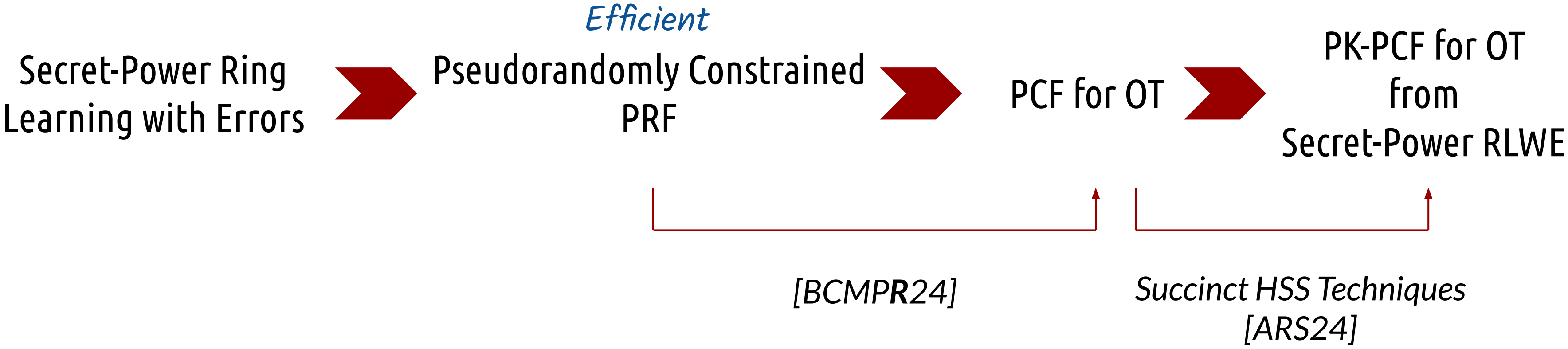
PCF for OT

PK-PCF for OT from Secret-Power RLWE

*[BCMP**R**24]*

Contributions

Efficient Public-Key PCF for OT Correlations from Lattices



Secret-Power Ring Learning with Errors

Efficient Pseudorandomly Constrained PRF

PCF for OT

PK-PCF for OT from Secret-Power RLWE

*[BCMP**R**24]*

Succinct HSS Techniques [ARS24]

HSS: Homomorphic Secret Sharing

Contributions

Efficient Public-Key PCF for OT Correlations from Lattices

In this talk

Secret-Power Ring Learning with Errors

Efficient

Pseudorandomly Constrained PRF

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PK-PCF for OT from Secret-Power RLWE

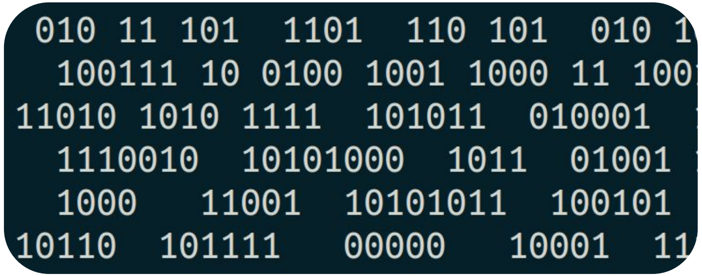
Pseudorandom Functions

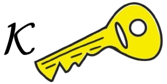
Pseudorandom Functions (PRFs) [GGM86]

Definition. Deterministic keyed functions indistinguishable from truly random functions.

$F : \mathcal{K} \times \mathcal{X} \to \mathcal{Y}$

Set of outputs with msk



Compute using **msk** $\xleftarrow{\$} \mathcal{K}$ 

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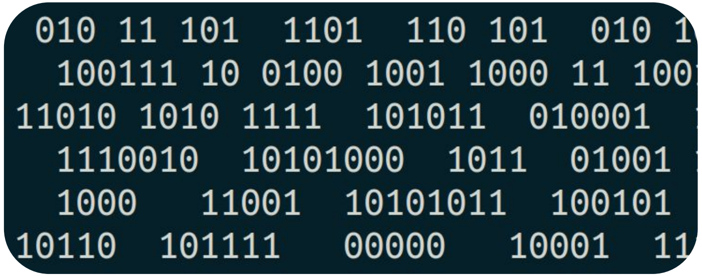
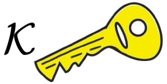


Compute using **msk** $\xleftarrow{\$} \mathcal{K}$



Set of outputs without msk

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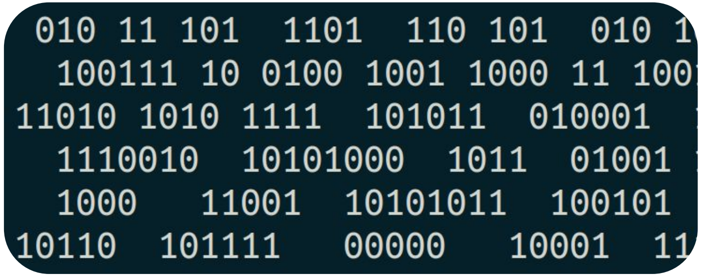

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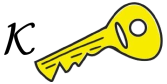
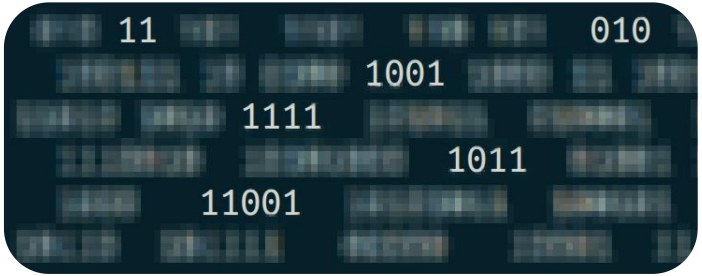
   100111 10 0100 1001 1000 11
11010 1010 1111  101011  010001
   1110010  10101000  1011  01001
   1000    11001  10101011  100101
10110  101111   00000   10001  11
```

Compute using **msk** $\xleftarrow{\$} \mathcal{K}$



Set of outputs without msk



with oracle queries on $\underline{\text{arbitrary}}$ / $\underline{\text{random}}$ inputs

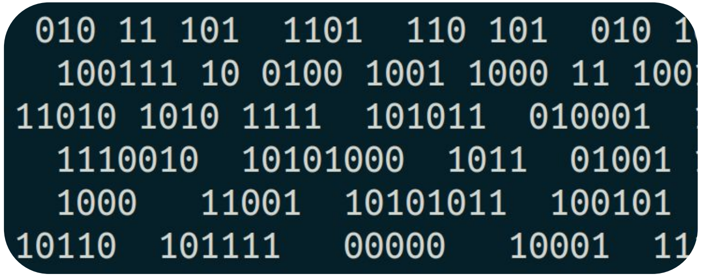
PRF   weak PRF

Constrained Pseudorandom Functions

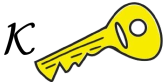
Constrained Pseudorandom Functions (CPRFs)[BW13,KPTZ13,BGI14]

Pseudorandom Functions *with constrained access to the evaluation.*

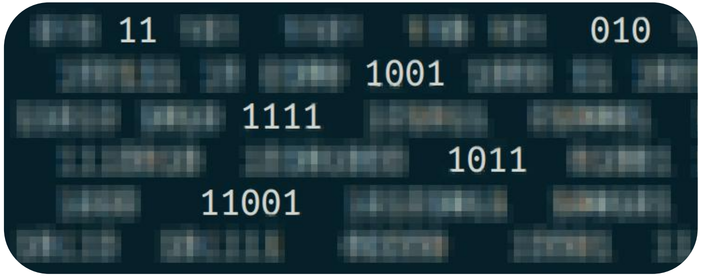
Set of outputs with msk



Compute using **msk** $\xleftarrow{\$} \mathcal{K}$



Set of outputs without msk



with oracle queries on arbitrary / random inputs

Constrained Pseudorandom Functions (CPRFs)[BW13,KPTZ13,BGI14]

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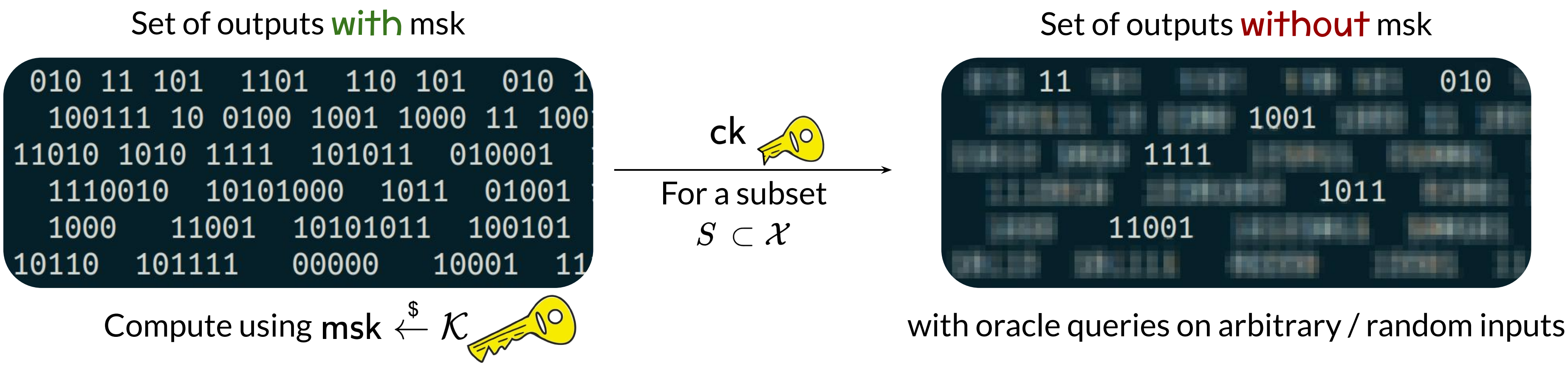


Set of outputs with msk

ck

For a subset $S \subset \mathcal{X}$

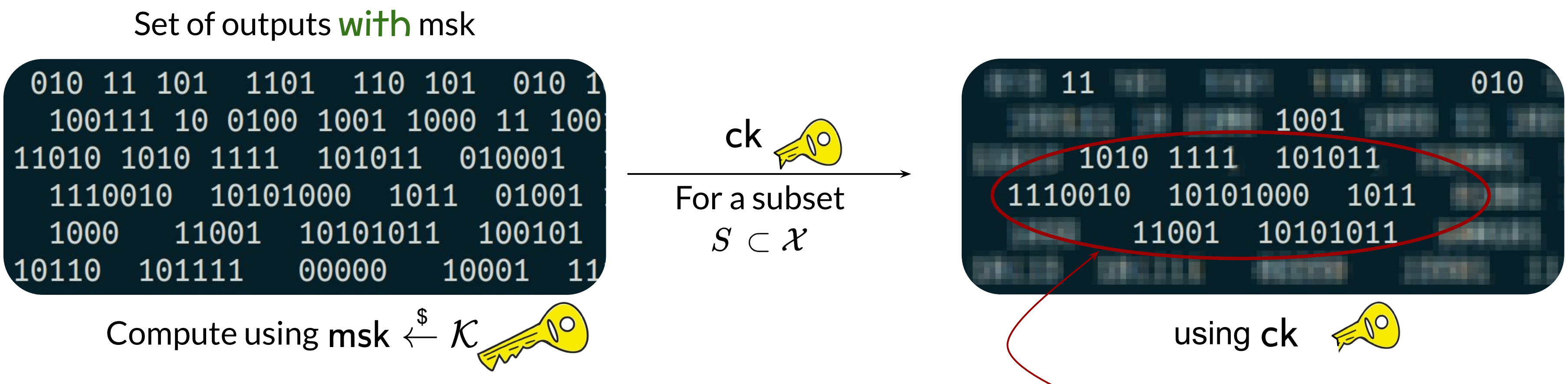
Set of outputs without msk

Compute using **msk** $\xleftarrow{\$} \mathcal{K}$

with oracle queries on arbitrary / random inputs

Constrained Pseudorandom Functions (CPRFs)[BW13,KPTZ13,BGI14]

Pseudorandom Functions *with constrained access to the evaluation.*



Set of outputs with msk

ck

For a subset

$S \subset \mathcal{X}$

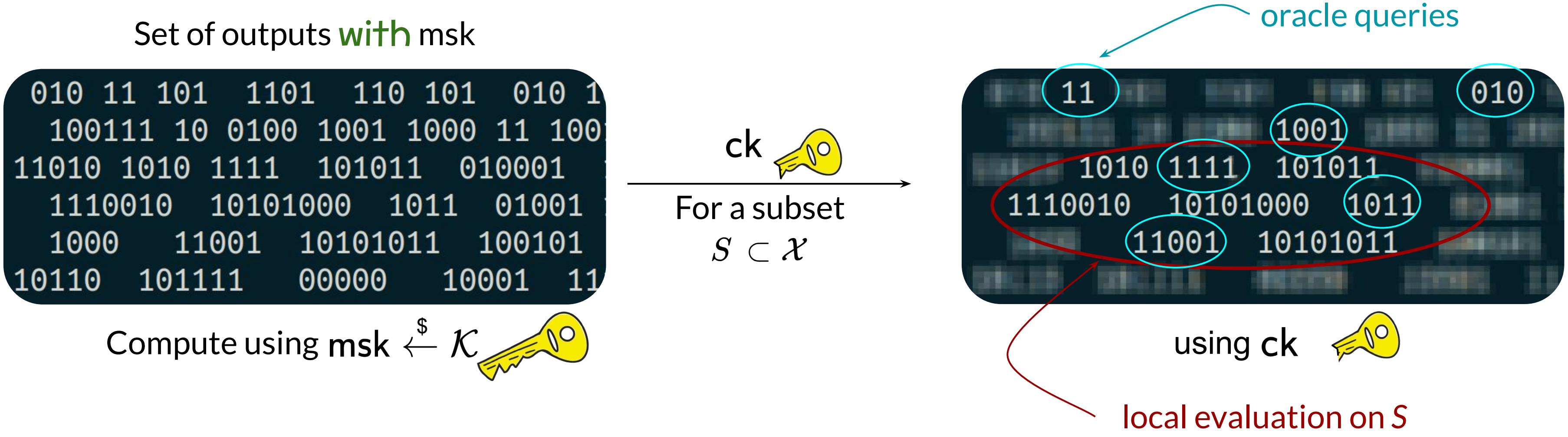
Compute using **msk** $\xleftarrow{\$} \mathcal{K}$

using **ck**

local evaluation on *S*

Constrained Pseudorandom Functions (CPRFs)[BW13,KPTZ13,BGI14]

Pseudorandom Functions *with constrained access to the evaluation.*



Set of outputs with msk

Compute using **msk** $\xleftarrow{\$} \mathcal{K}$

ck

For a subset $S \subset \mathcal{X}$

oracle queries

using **ck**

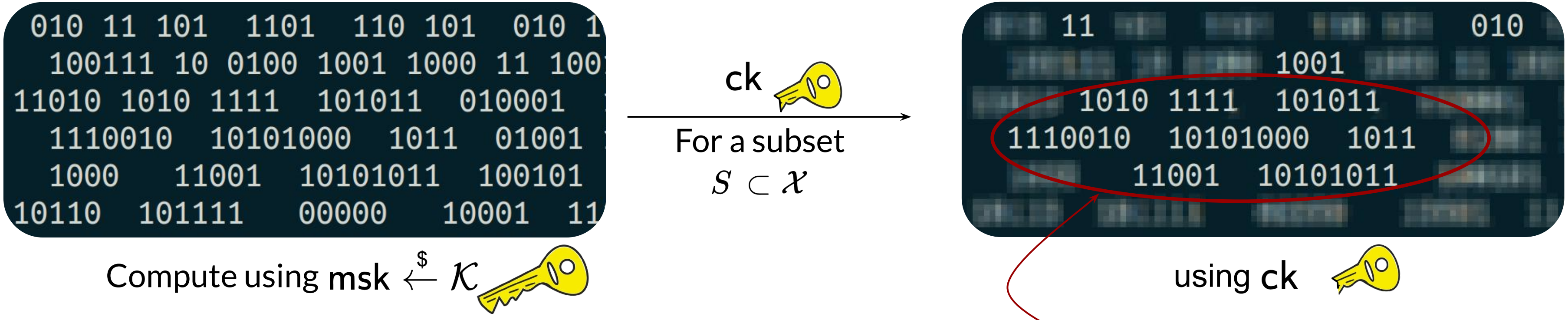
local evaluation on *S*

Constrained Pseudorandom Functions (CPRFs)[BW13,KPTZ13,BGI14]

Pseudorandom Functions *with constrained access to the evaluation.*

ck_S "=" msk ***only*** for all $x \in S$

Set of outputs with msk



ck

For a subset

$S \subset \mathcal{X}$

Compute using **msk** $\xleftarrow{\$} \mathcal{K}$

using **ck**

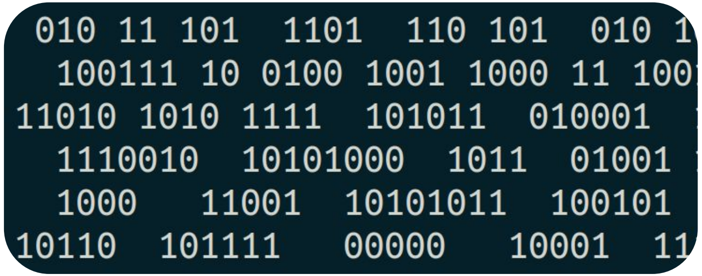
local evaluation on *S*

Constrained Pseudorandom Functions (CPRFs)[BW13,KPTZ13,BGI14]

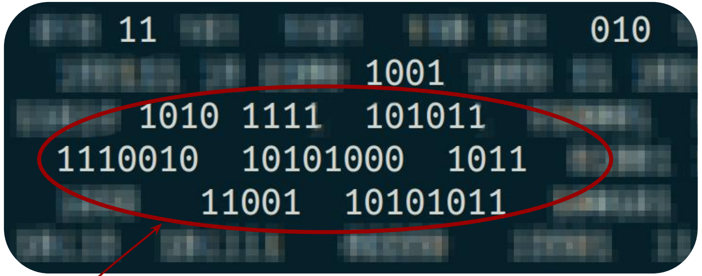
Pseudorandom Functions *with constrained access to the evaluation.*

- Every predicate $F : \mathcal{X} \to \{0,1\}$ defines a subset $S_F = \{x \in \mathcal{X} : F(x) = 0\}$

Set of outputs with msk



Compute using **msk** $\overset{\$}{\leftarrow} \mathcal{K}$



using $\textbf{ck}_F$

local evaluation on S_F

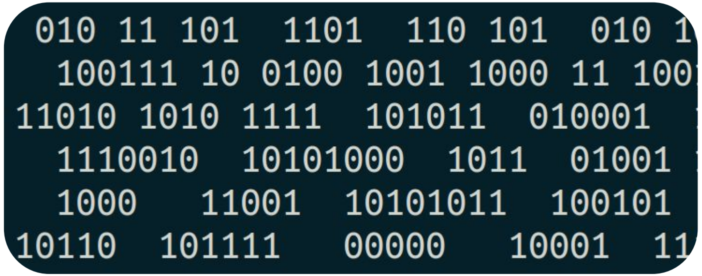
Constrained Pseudorandom Functions (CPRFs)[BW13,KPTZ13,BGI14]

Pseudorandom Functions *with constrained access to the evaluation.*

- Every predicate $\underbrace{F : \mathcal{X} \to \{0, 1\}}_{}$ defines a subset $S_F = \{x \in \mathcal{X} : F(x) = 0\}$

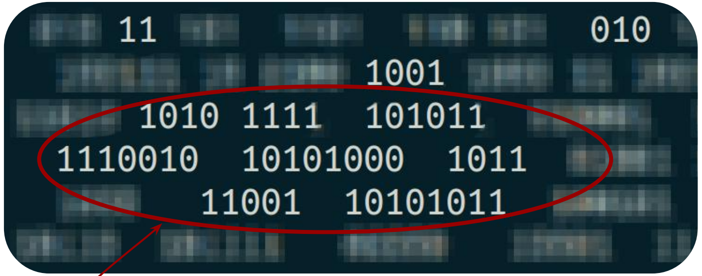
(w)PRF ⤳ Pseudorandomly Constrained PRF

Set of outputs with msk



```
010 11 101  1101  110 101  010 1
 100111 10 0100 1001 1000 11 100
11010 1010 1111  101011  010001
 1110010  10101000  1011  01001
 1000   11001  10101011  100101
10110  101111   00000   10001  11
```

Compute using **msk** $\xleftarrow{\$} \mathcal{K}$



```
  11                        010
           1001
     1010 1111  101011
1110010  10101000  1011
        11001  10101011
```

using $\mathbf{ck}_F$

local evaluation on S_F

Pseudorandom Correlation Functions
for
Oblivious Transfer

from pseudorandomly constrained PRFs

PCF for OT from Pseudorandomly Constrained PRFs [BCMPR24]

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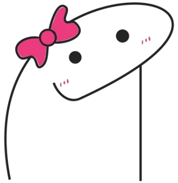
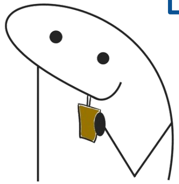
- wPRF $F_{\mathsf{k}} : \mathcal{X} \rightarrow \{0,1\}$

PCF for OT from Pseudorandomly Constrained PRFs [BCMPR24]

- wPRF $F_{\mathsf{k}} : \mathcal{X} \rightarrow \{0,1\}$
- CPRF for F_{k} and $\overline{F_{\mathsf{k}}}$





PCF for OT from Pseudorandomly Constrained PRFs [BCMPR24]

- wPRF $F_{\mathsf{k}} : \mathcal{X} \to \{0,1\}$
- CPRF for F_{k} and $\overline{F_{\mathsf{k}}}$

can generate a **ck** for either:

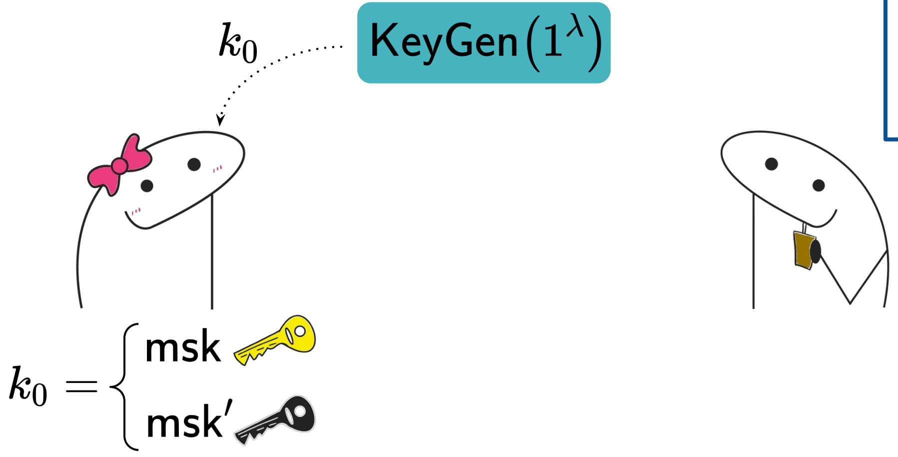
-any **x** s.t. **F_k(x)=0**

or

-any **x** s.t. **F_k(x)=1**

PCF for OT from Pseudorandomly Constrained PRFs [BCMPR24]

- wPRF $F_\mathsf{k} : \mathcal{X} \to \{0,1\}$
- CPRF for F_k and $\overline{F_\mathsf{k}}$



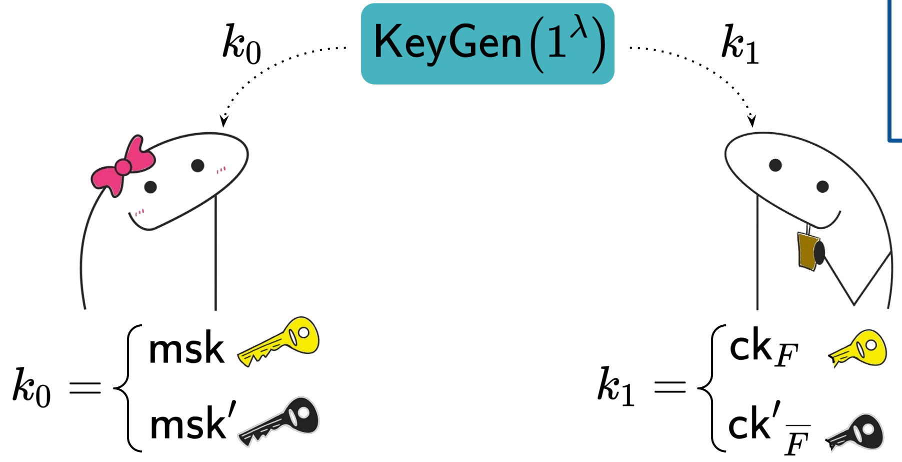
k_0

$\mathsf{KeyGen}(1^\lambda)$

$$k_0 = \begin{cases} \mathsf{msk} \\ \mathsf{msk}' \end{cases}$$

PCF for OT from Pseudorandomly Constrained PRFs [BCMPR24]

- wPRF $F_{\mathsf{k}} : \mathcal{X} \rightarrow \{0,1\}$
- CPRF for F_{k} and $\overline{F_{\mathsf{k}}}$



KeyGen(1^λ)

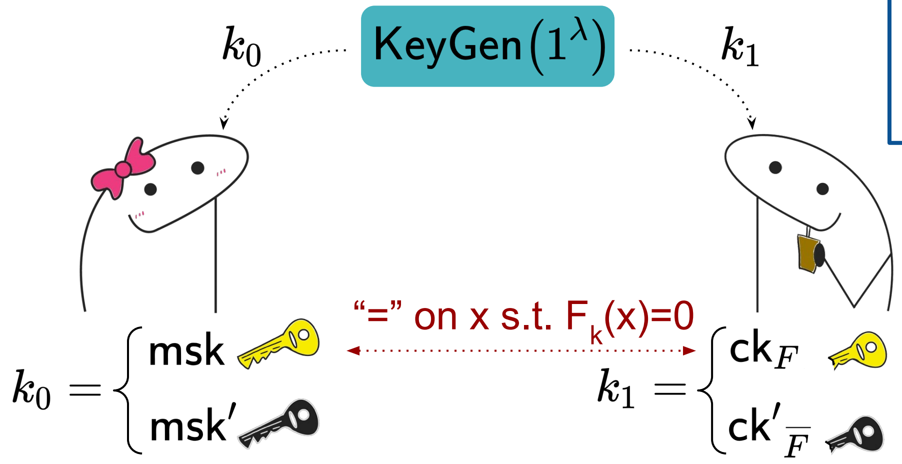
k_0 k_1

$$k_0 = \begin{cases} \mathsf{msk} \\ \mathsf{msk}' \end{cases} \qquad k_1 = \begin{cases} \mathsf{ck}_F \\ \mathsf{ck}'_{\overline{F}} \end{cases}, \; \mathsf{k}$$

(wPRF key)

PCF for OT from Pseudorandomly Constrained PRFs [BCMPR24]

- wPRF $F_k : \mathcal{X} \to \{0,1\}$
- CPRF for F_k and $\overline{F_k}$



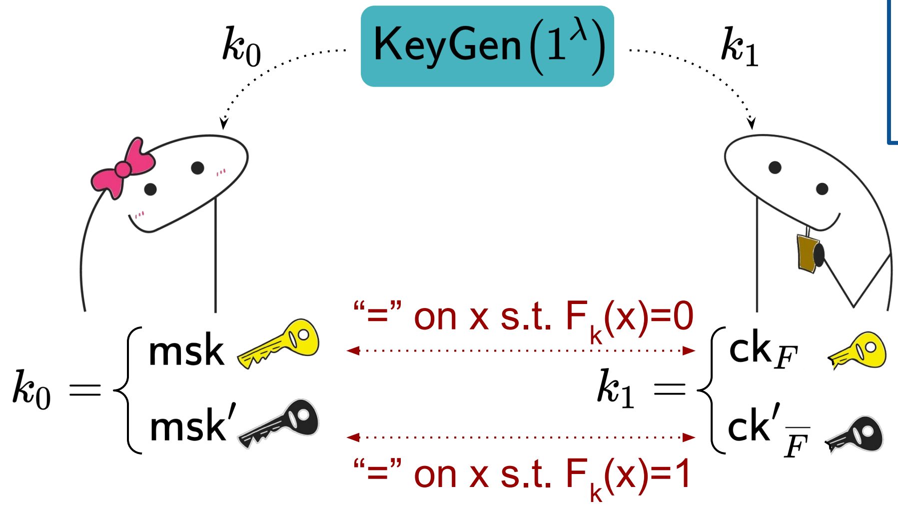
$$k_0 = \begin{cases} \text{msk} \\ \text{msk}' \end{cases} \qquad k_1 = \begin{cases} \text{ck}_F \\ \text{ck}'_{\overline{F}} \end{cases}$$

, k (wPRF key)

PCF for OT from Pseudorandomly Constrained PRFs [BCMPR24]

- wPRF $F_k : \mathcal{X} \to \{0,1\}$

- CPRF for F_k and $\overline{F_k}$



$\mathsf{KeyGen}(1^\lambda)$

k_0 k_1

$k_0 = \begin{cases} \mathsf{msk} \\ \mathsf{msk}' \end{cases}$

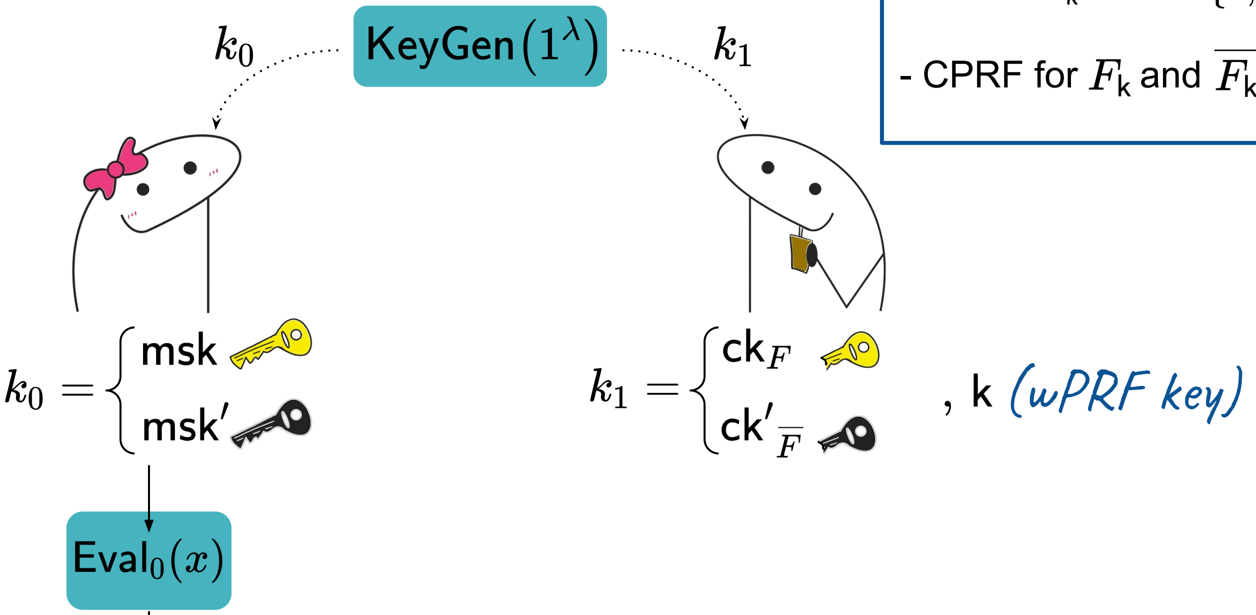
"=" on x s.t. $F_k(x)=0$

"=" on x s.t. $F_k(x)=1$

$k_1 = \begin{cases} \mathsf{ck}_F \\ \mathsf{ck}'_{\overline{F}} \end{cases}$, k *(wPRF key)*

PCF for OT from Pseudorandomly Constrained PRFs [BCMPR24]

- wPRF $F_{\mathsf{k}} : \mathcal{X} \rightarrow \{0,1\}$
- CPRF for F_{k} and $\overline{F_{\mathsf{k}}}$



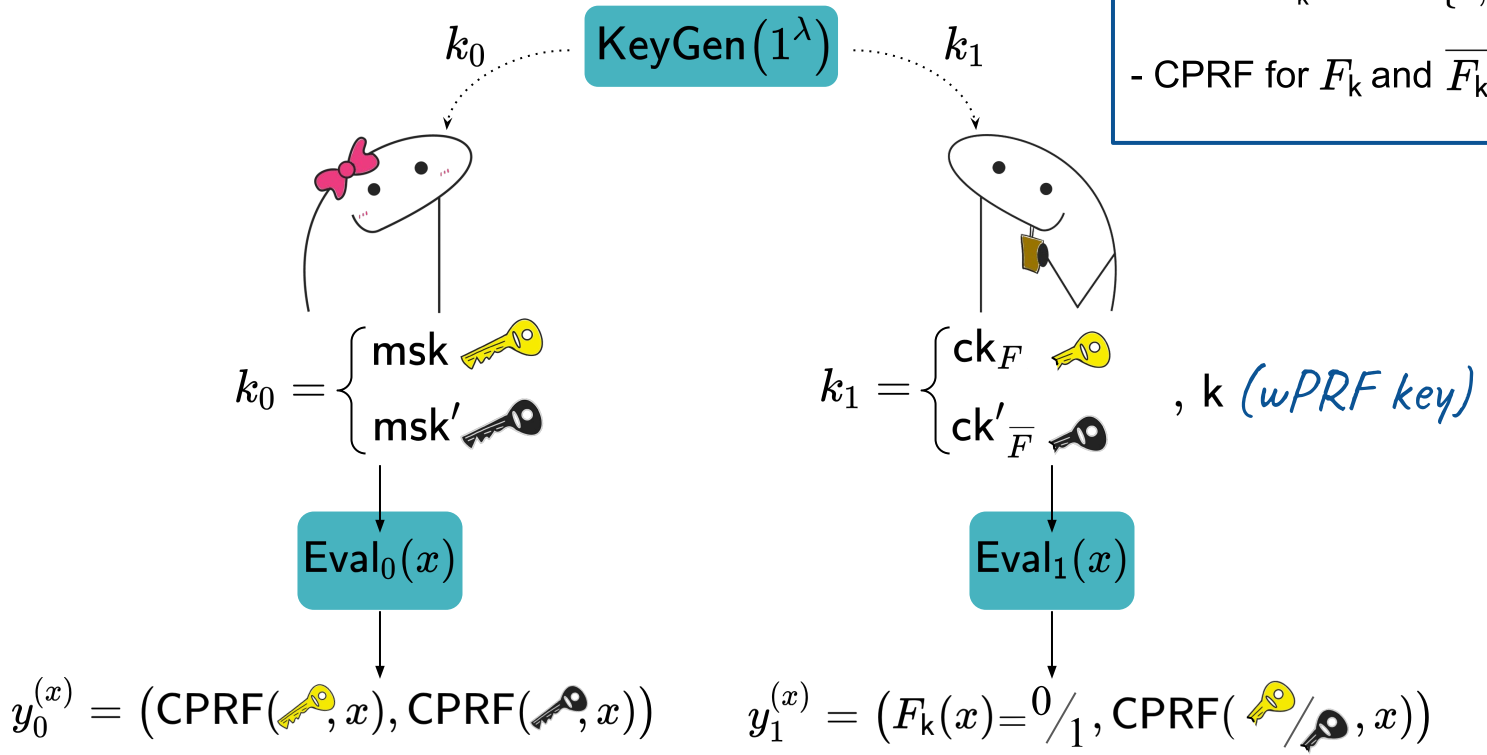
$k_0 = \begin{cases} \mathsf{msk} \text{ 🔑} \\ \mathsf{msk}' \text{ 🗝} \end{cases}$

$k_1 = \begin{cases} \mathsf{ck}_F \text{ 🔑} \\ \mathsf{ck}'_{\overline{F}} \text{ 🗝} \end{cases}$, k (wPRF key)

$y_0^{(x)} = (\mathsf{CPRF}(\text{🔑}, x), \mathsf{CPRF}(\text{🗝}, x))$

PCF for OT from Pseudorandomly Constrained PRFs [BCMPR24]

- wPRF $F_{\mathsf{k}} : \mathcal{X} \to \{0,1\}$
- CPRF for F_{k} and $\overline{F_{\mathsf{k}}}$



KeyGen(1^λ)

k_0

k_1

$$k_0 = \begin{cases} \mathsf{msk}\ \text{🔑} \\ \mathsf{msk}'\ \text{🗝} \end{cases}$$

$$k_1 = \begin{cases} \mathsf{ck}_F\ \text{🔑} \\ \mathsf{ck}'_{\overline{F}}\ \text{🗝} \end{cases}, \ \mathsf{k}$$ (wPRF key)

Eval$_0(x)$

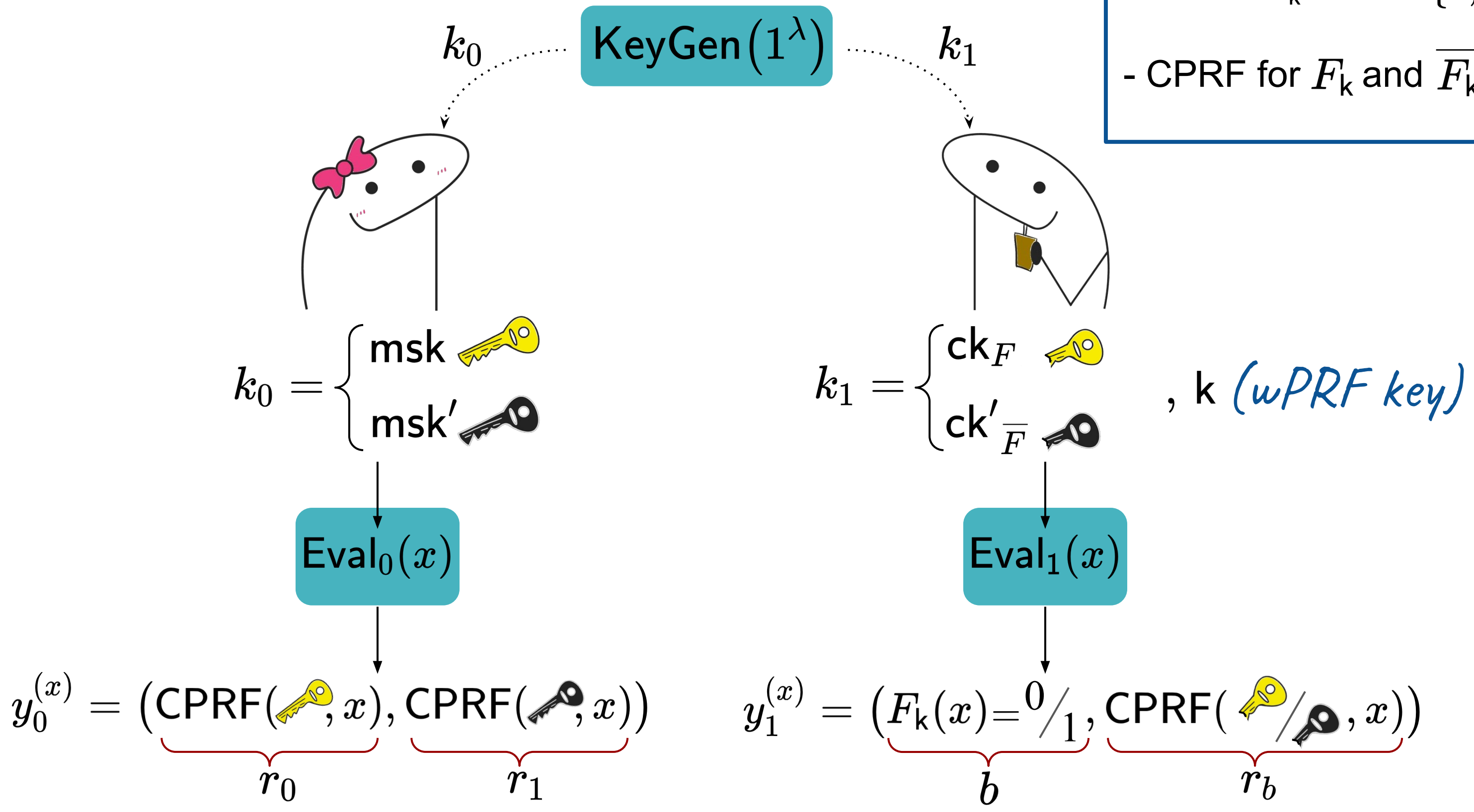
Eval$_1(x)$

$$y_0^{(x)} = (\mathsf{CPRF}(\text{🔑}, x), \mathsf{CPRF}(\text{🗝}, x))$$

$$y_1^{(x)} = (F_{\mathsf{k}}(x) = {}^0/_1, \mathsf{CPRF}(\text{🔑}/\text{🗝}, x))$$

PCF for OT from Pseudorandomly Constrained PRFs [BCMPR24]

- wPRF $F_k : \mathcal{X} \to \{0,1\}$
- CPRF for F_k and $\overline{F_k}$



$$k_0 = \begin{cases} \text{msk } 🔑 \\ \text{msk}' \ 🗝 \end{cases}$$

$$k_1 = \begin{cases} \text{ck}_F \ 🔑 \\ \text{ck}'_{\overline{F}} \ 🗝 \end{cases}, \text{ k}$$ (wPRF key)

$$y_0^{(x)} = (\underbrace{\text{CPRF}(🔑, x)}_{r_0}, \underbrace{\text{CPRF}(🗝, x)}_{r_1})$$

$$y_1^{(x)} = (\underbrace{F_k(x) = {}^0/_1}_{b}, \underbrace{\text{CPRF}(🔑/🗝, x)}_{r_b})$$

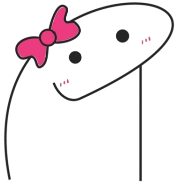
PCF for OT from Pseudorandomly Constrained PRFs [BCMPR24]

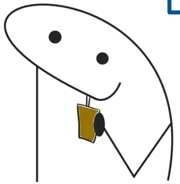
- wPRF $F_k : \mathcal{X} \to \{0,1\}$
- CPRF for F_k and $\overline{F_k}$

$\mathsf{KeyGen}(1^\lambda)$

k_0

k_1





$$k_0 = \begin{cases} \mathsf{msk}\ \text{🔑} \\ \mathsf{msk}'\ \text{🗝} \end{cases}$$

$$k_1 = \begin{cases} \mathsf{ck}_F\ \text{🔑} \\ \mathsf{ck}'_{\overline{F}}\ \text{🗝} \end{cases}, \ \mathsf{k}$$ (wPRF key)

$\mathsf{Eval}_0(x)$

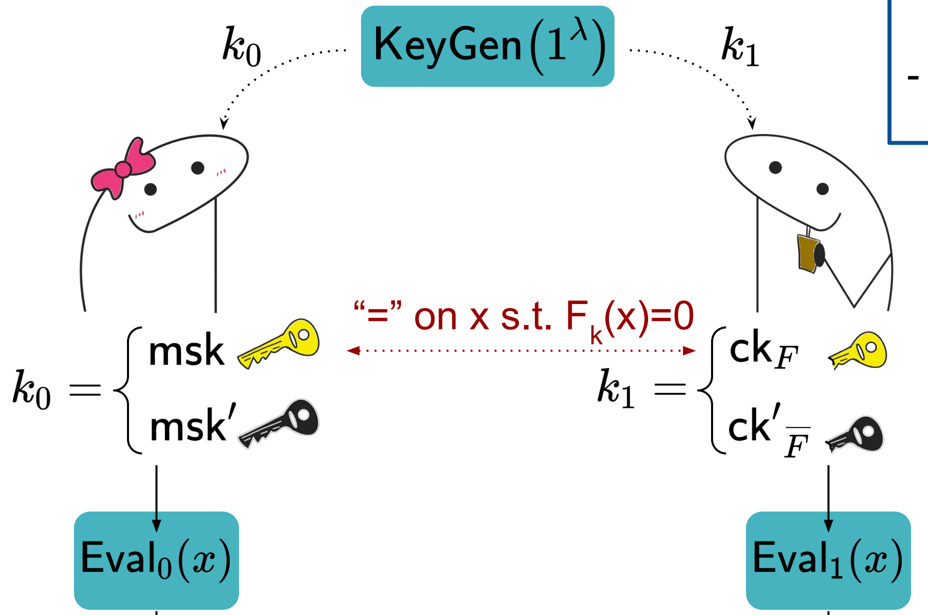
$\mathsf{Eval}_1(x)$

$$y_0^{(x)} = (\underbrace{\mathsf{CPRF}(\text{🔑}, x)}_{r_0}, \underbrace{\mathsf{CPRF}(\text{🗝}, x)}_{r_1})$$

$$y_1^{(x)} = (F_k(x) = 0, \mathsf{CPRF}(\text{🔑}, x))$$

PCF for OT from Pseudorandomly Constrained PRFs [BCMPR24]

- wPRF $F_k : \mathcal{X} \to \{0,1\}$
- CPRF for F_k and $\overline{F_k}$



$k_0 = \begin{cases} \text{msk 🔑} \\ \text{msk}' \text{ 🗝} \end{cases}$

"=" on x s.t. $F_k(x)=0$

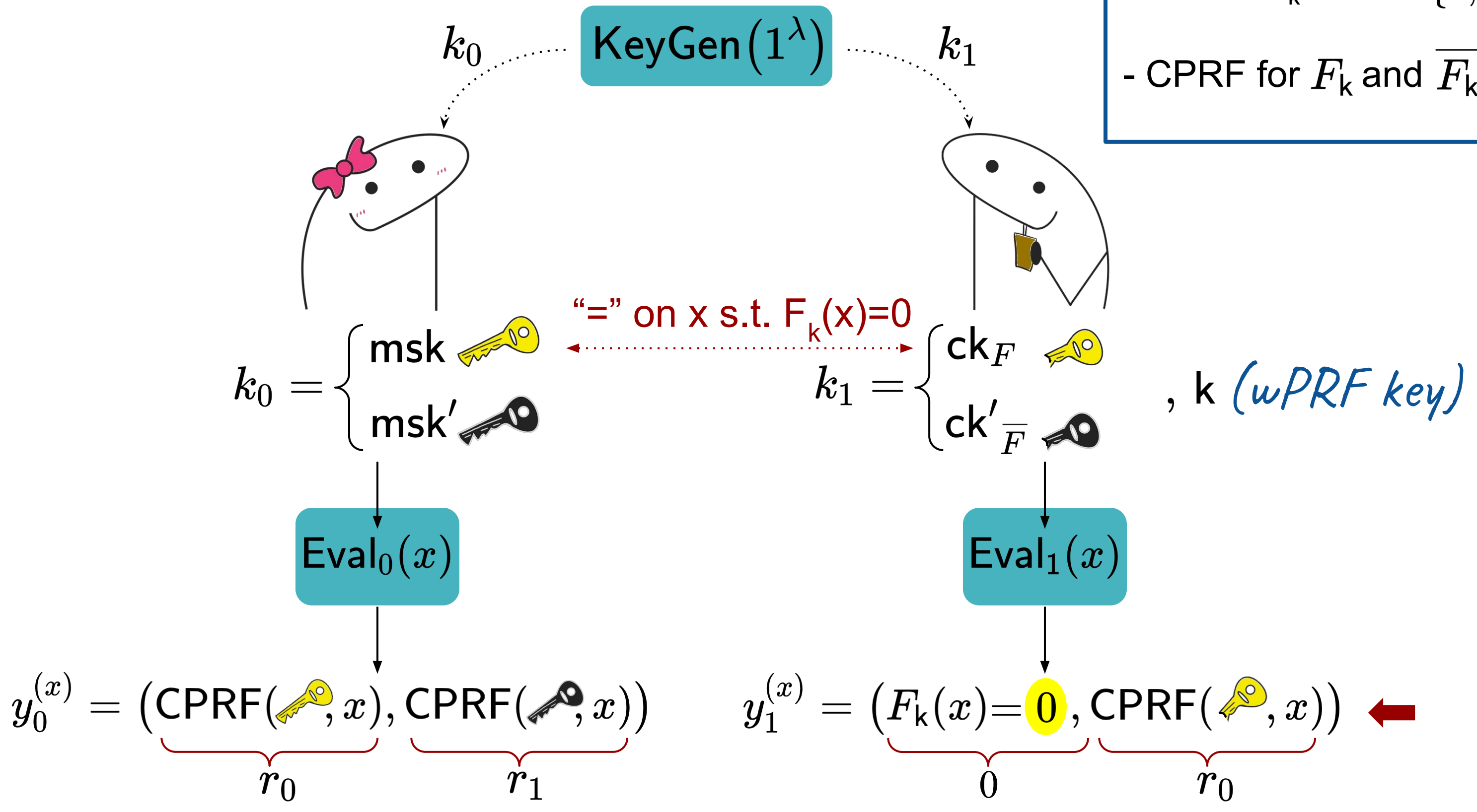
$k_1 = \begin{cases} \text{ck}_F \text{ 🔑} \\ \text{ck}'_{\overline{F}} \text{ 🗝} \end{cases}$, k *(wPRF key)*

$$y_0^{(x)} = (\underbrace{\text{CPRF}(\text{🔑}, x)}_{r_0}, \underbrace{\text{CPRF}(\text{🗝}, x)}_{r_1})$$

$$y_1^{(x)} = (F_k(x) = 0, \text{CPRF}(\text{🔑}, x))$$

PCF for OT from Pseudorandomly Constrained PRFs [BCMPR24]

- wPRF $F_k : \mathcal{X} \to \{0,1\}$
- CPRF for F_k and $\overline{F_k}$



$\mathsf{KeyGen}(1^\lambda)$

k_0 k_1

“=” on x s.t. $F_k(x)=0$

$k_0 = \begin{cases} \mathsf{msk} \text{ 🔑} \\ \mathsf{msk}' \text{ 🗝} \end{cases}$

$k_1 = \begin{cases} \mathsf{ck}_F \text{ 🔑} \\ \mathsf{ck}'_{\overline{F}} \text{ 🗝} \end{cases}$, k (wPRF key)

$\mathsf{Eval}_0(x)$

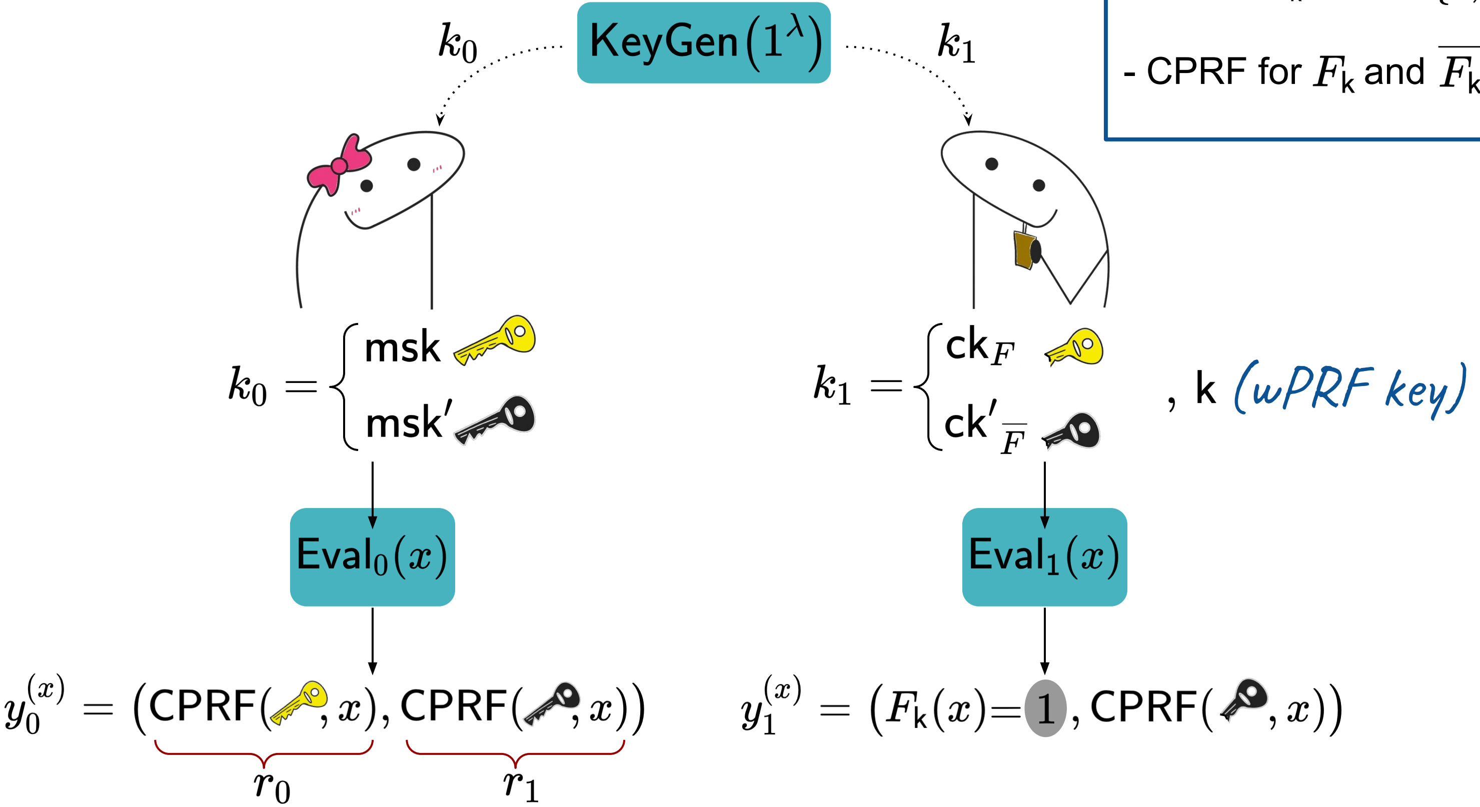
$\mathsf{Eval}_1(x)$

$$y_0^{(x)} = (\underbrace{\mathsf{CPRF}(\text{🔑}, x)}_{r_0}, \underbrace{\mathsf{CPRF}(\text{🗝}, x)}_{r_1})$$

$$y_1^{(x)} = (\underbrace{F_k(x) = 0}_{0}, \underbrace{\mathsf{CPRF}(\text{🔑}, x)}_{r_0})$$ ⬅

PCF for OT from Pseudorandomly Constrained PRFs [BCMPR24]

- wPRF $F_k : \mathcal{X} \to \{0,1\}$
- CPRF for F_k and $\overline{F_k}$



KeyGen(1^λ)

k_0 k_1

$$k_0 = \begin{cases} \text{msk } \text{🔑} \\ \text{msk}' \text{ 🗝} \end{cases}$$

$$k_1 = \begin{cases} \text{ck}_F \text{ 🔑} \\ \text{ck}'_{\overline{F}} \text{ 🗝} \end{cases}$$, k (wPRF key)

Eval$_0(x)$

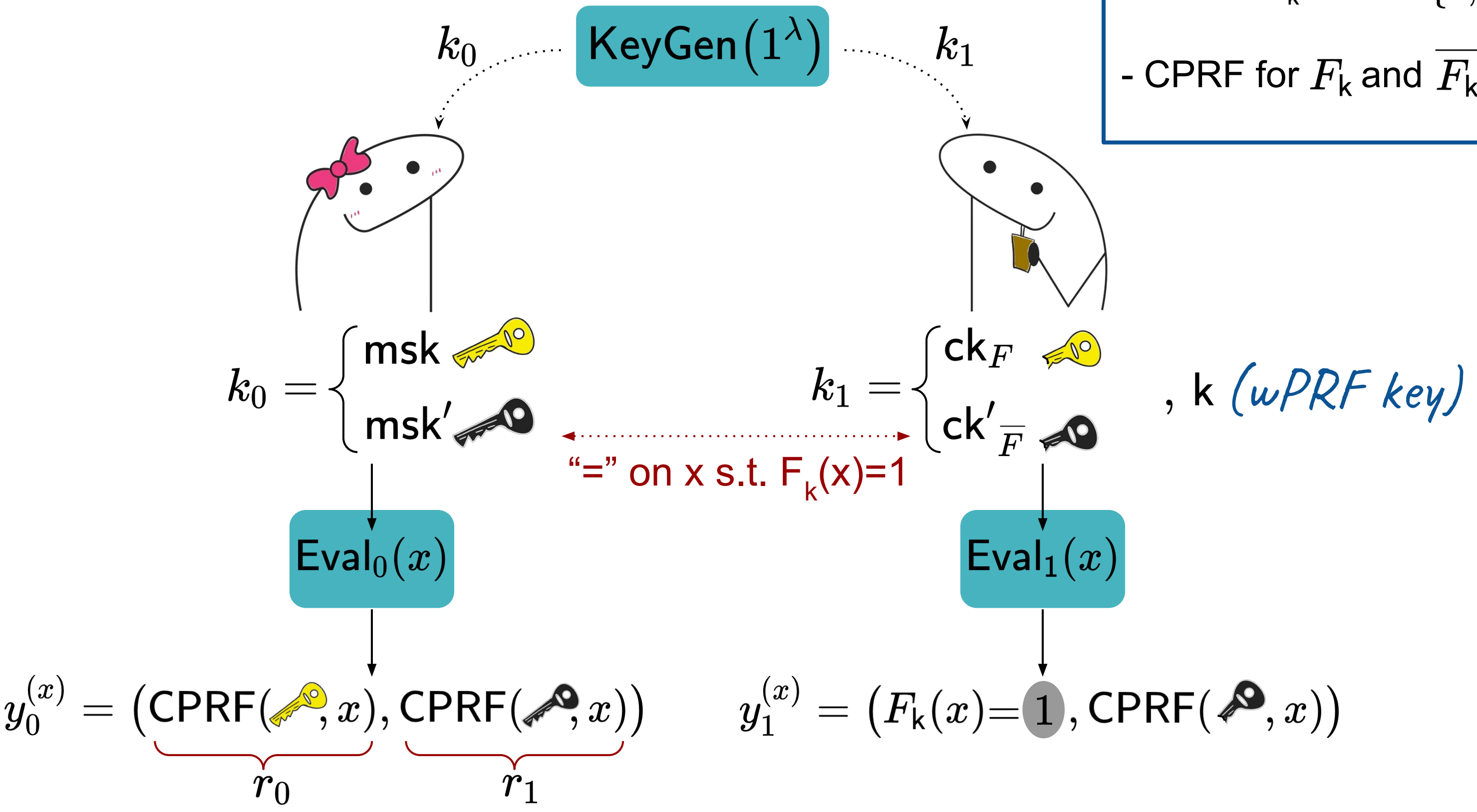
Eval$_1(x)$

$$y_0^{(x)} = (\underbrace{\text{CPRF}(\text{🔑}, x)}_{r_0}, \underbrace{\text{CPRF}(\text{🗝}, x)}_{r_1})$$

$$y_1^{(x)} = (F_k(x) = 1, \text{CPRF}(\text{🗝}, x))$$

PCF for OT from Pseudorandomly Constrained PRFs [BCMPR24]

- wPRF $F_{\mathsf{k}} : \mathcal{X} \to \{0,1\}$
- CPRF for F_{k} and $\overline{F_{\mathsf{k}}}$



$\mathsf{KeyGen}(1^\lambda)$

k_0 k_1

$$k_0 = \begin{cases} \mathsf{msk} \; 🔑 \\ \mathsf{msk}' \; 🗝 \end{cases}$$

$$k_1 = \begin{cases} \mathsf{ck}_F \; 🔑 \\ \mathsf{ck}'_{\overline{F}} \; 🗝 \end{cases}, \; \mathsf{k}$$ (wPRF key)

“=” on x s.t. $F_k(x)=1$

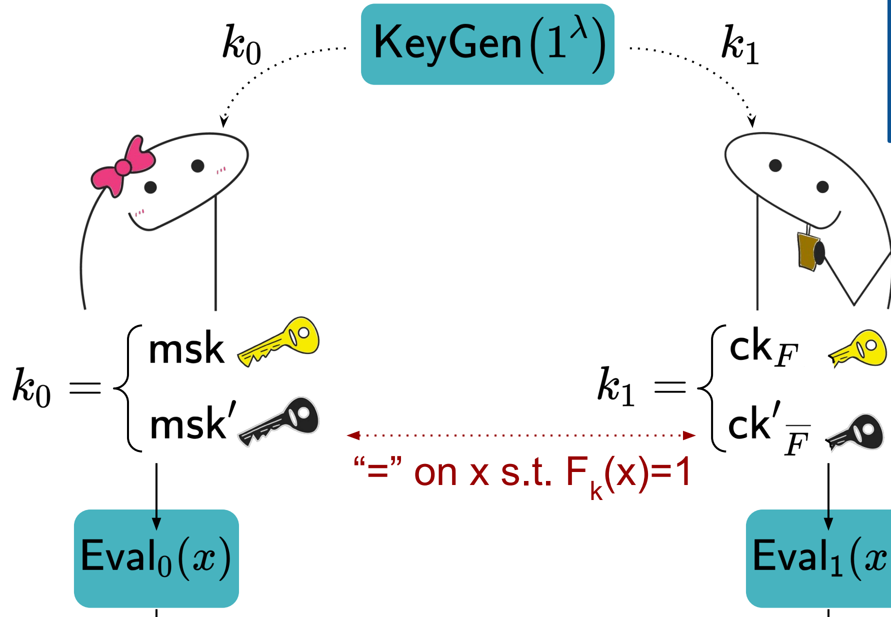
$\mathsf{Eval}_0(x)$ $\mathsf{Eval}_1(x)$

$$y_0^{(x)} = (\underbrace{\mathsf{CPRF}(🔑, x)}_{r_0}, \underbrace{\mathsf{CPRF}(🗝, x)}_{r_1})$$

$$y_1^{(x)} = (F_{\mathsf{k}}(x) = 1, \mathsf{CPRF}(🗝, x))$$

PCF for OT from Pseudorandomly Constrained PRFs [BCMPR24]

- wPRF $F_k : \mathcal{X} \to \{0,1\}$
- CPRF for F_k and $\overline{F_k}$



KeyGen(1^λ)

k_0

k_1

$k_0 = \{$ msk 🔑, msk$'$ 🗝 $\}$

$k_1 = \{$ ck$_F$ 🔑, ck$'_{\overline{F}}$ 🗝 $\}$, k *(wPRF key)*

"=" on x s.t. $F_k(x)$=1

Eval$_0(x)$

Eval$_1(x)$

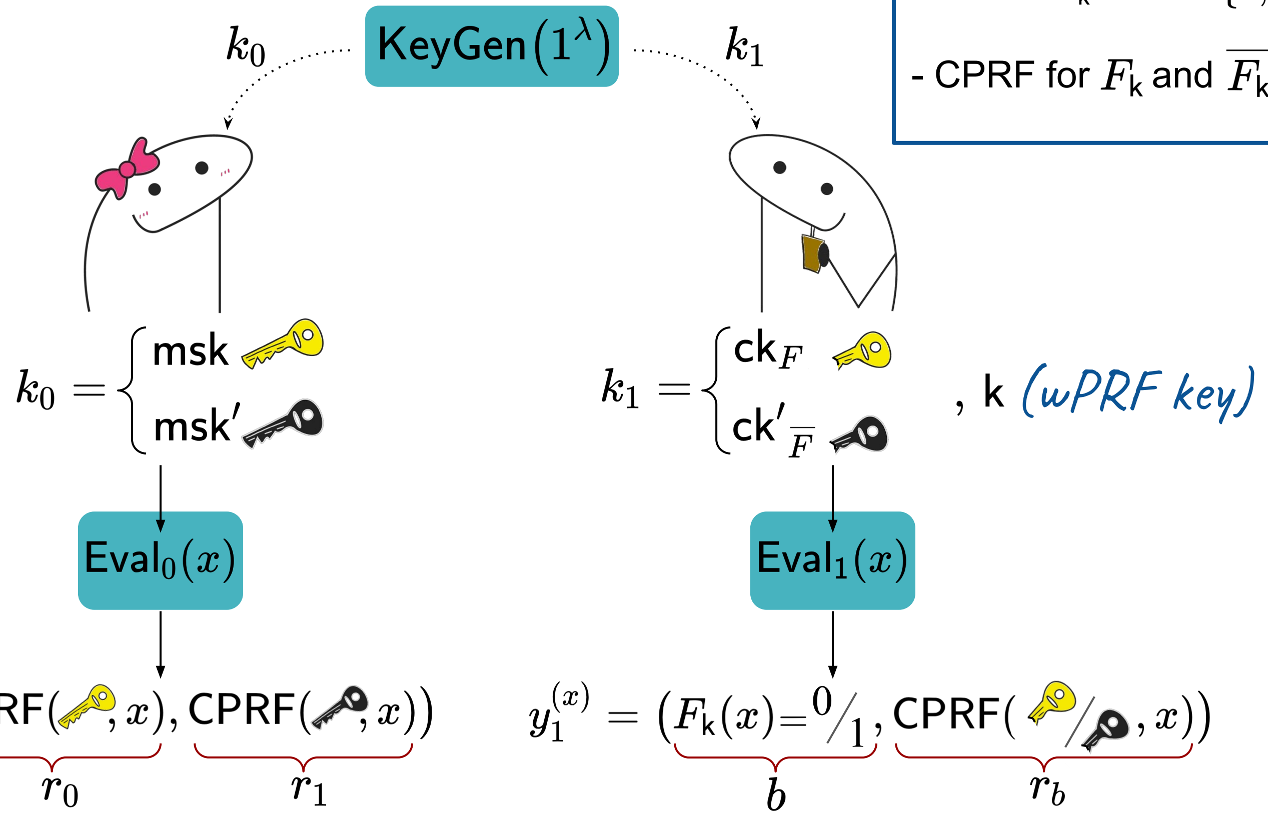
$$y_0^{(x)} = (\underbrace{\text{CPRF}(\text{🔑}, x)}_{r_0}, \underbrace{\text{CPRF}(\text{🗝}, x)}_{r_1})$$

$$y_1^{(x)} = (\underbrace{F_k(x) = \boxed{1}}_{1}, \underbrace{\text{CPRF}(\text{🗝}, x)}_{r_1})$$

PCF for OT from Pseudorandomly Constrained PRFs [BCMPR24]

OT-correlated ✓

- wPRF $F_k : \mathcal{X} \to \{0,1\}$
- CPRF for F_k and $\overline{F_k}$



KeyGen(1^λ)

k_0 k_1

$$k_0 = \begin{cases} \text{msk} \ 🔑 \\ \text{msk}' \ 🔑 \end{cases}$$

$$k_1 = \begin{cases} \text{ck}_F \ 🔑 \\ \text{ck}'_{\overline{F}} \ 🔑 \end{cases}, \text{k}$$

(wPRF key)

Eval$_0(x)$

Eval$_1(x)$

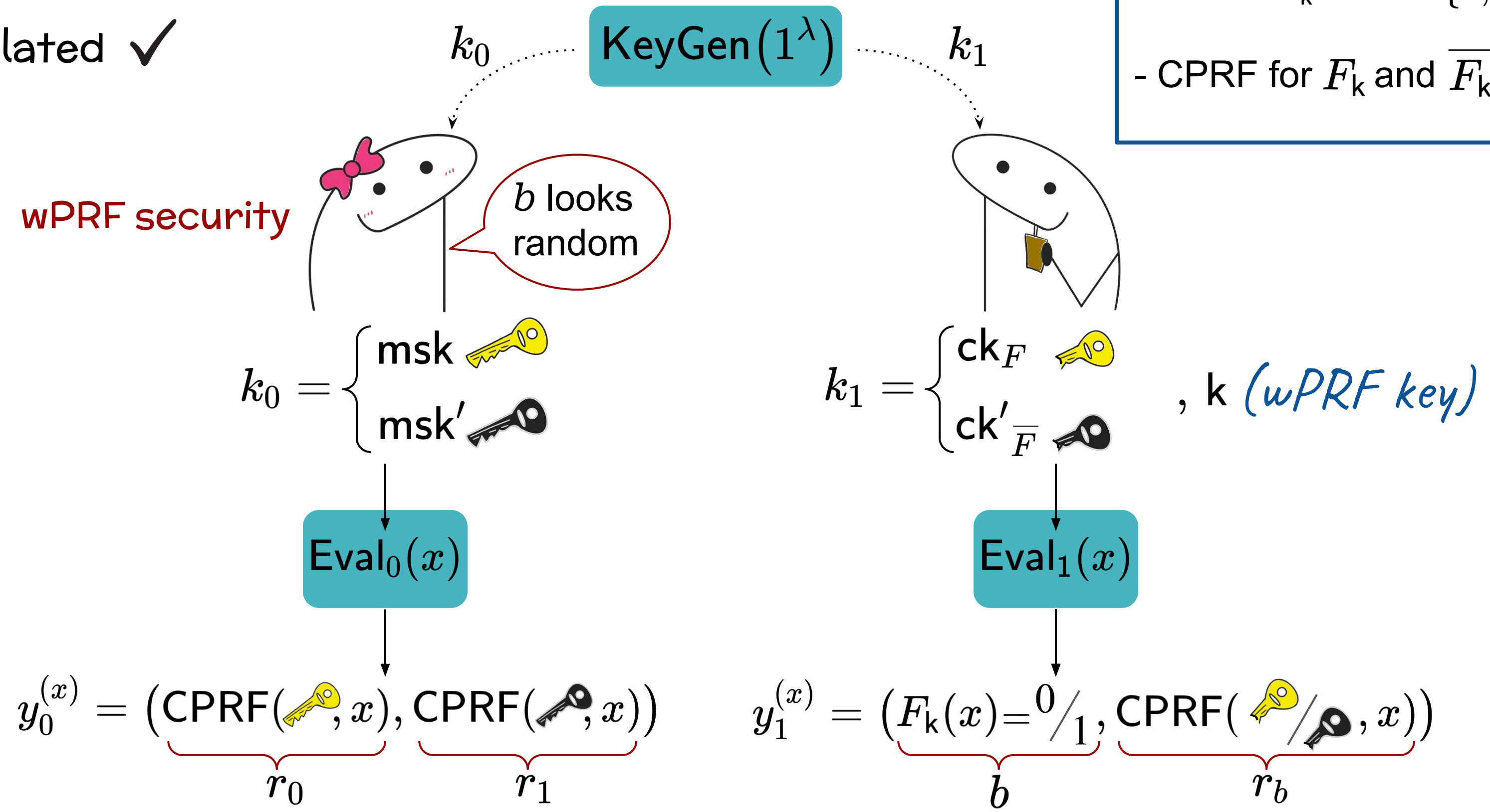
$$y_0^{(x)} = (\underbrace{\text{CPRF}(🔑, x)}_{r_0}, \underbrace{\text{CPRF}(🔑, x)}_{r_1})$$

$$y_1^{(x)} = (\underbrace{F_k(x) = {}^0/_1}_{b}, \underbrace{\text{CPRF}(🔑/🔑, x)}_{r_b})$$

PCF for OT from Pseudorandomly Constrained PRFs [BCMPR24]

- wPRF $F_k : \mathcal{X} \to \{0,1\}$
- CPRF for F_k and $\overline{F_k}$

OT-correlated ✓



wPRF security

b looks random

$\text{KeyGen}(1^\lambda)$

k_0 k_1

$$k_0 = \begin{cases} \text{msk} \ 🔑 \\ \text{msk}' \ 🗝 \end{cases}$$

$$k_1 = \begin{cases} \text{ck}_F \ 🔑 \\ \text{ck}'_{\overline{F}} \ 🗝 \end{cases}, \ \text{k}$$ (wPRF key)

$\text{Eval}_0(x)$

$\text{Eval}_1(x)$

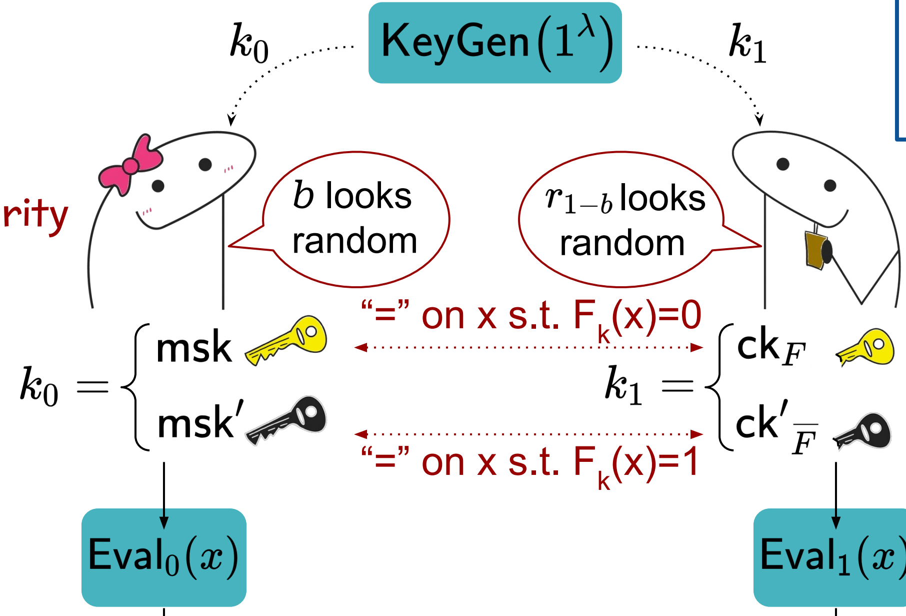
$$y_0^{(x)} = (\underbrace{\text{CPRF}(🔑, x)}_{r_0}, \underbrace{\text{CPRF}(🗝, x)}_{r_1})$$

$$y_1^{(x)} = (\underbrace{F_k(x) = {}^0/_1}_{b}, \underbrace{\text{CPRF}(🔑/🗝, x)}_{r_b})$$

PCF for OT from Pseudorandomly Constrained PRFs [BCMPR24]

- wPRF $F_k : \mathcal{X} \to \{0,1\}$
- CPRF for F_k and $\overline{F_k}$

OT-correlated ✓



$\text{KeyGen}(1^\lambda)$

k_0 k_1

wPRF security

b looks random

r_{1-b} looks random

CPRF security

"=" on x s.t. $F_k(x)=0$

$$k_0 = \begin{cases} \text{msk } 🔑 \\ \text{msk}' \ 🗝 \end{cases} \qquad k_1 = \begin{cases} \text{ck}_F \ 🔑 \\ \text{ck}'_{\overline{F}} \ 🗝 \end{cases}, \text{ k } (wPRF\ key)$$

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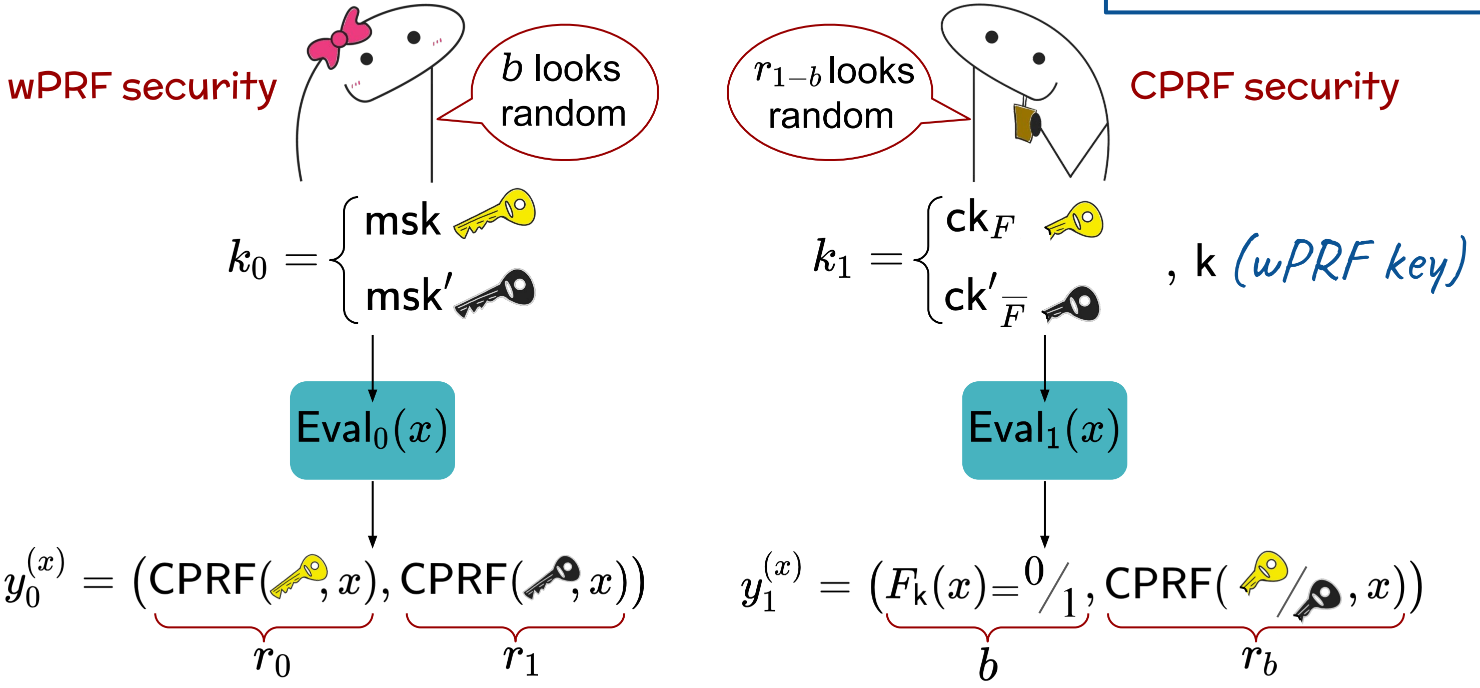

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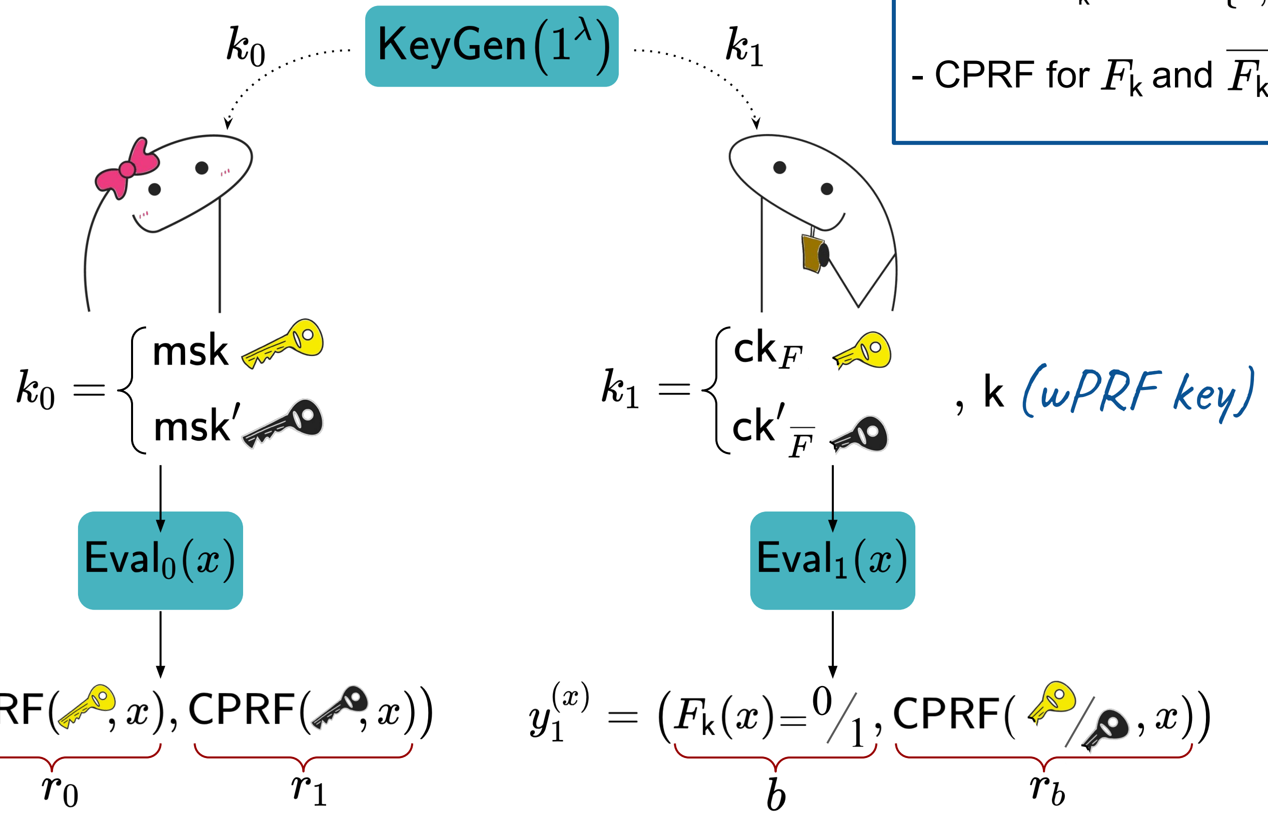
Precomputable



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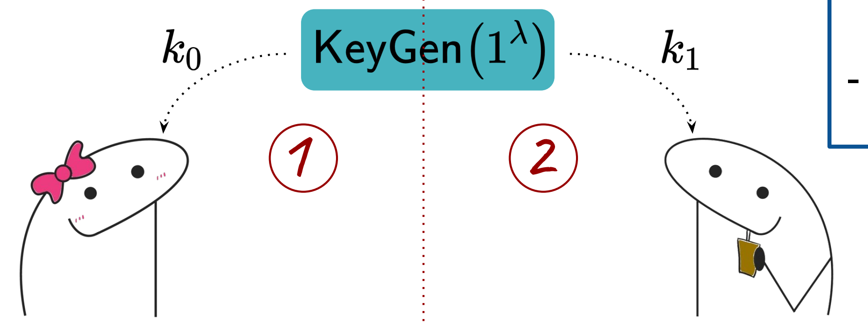

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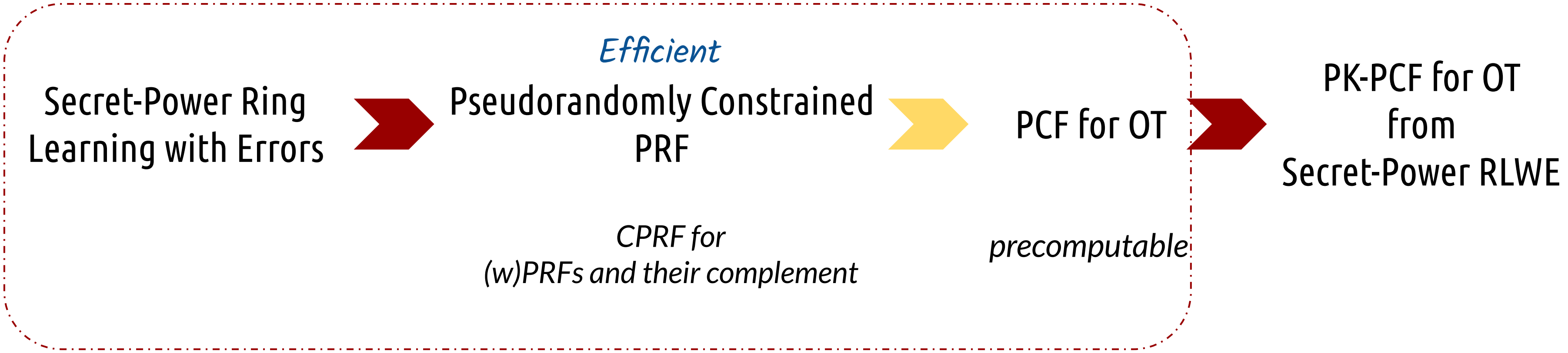
Contributions

Efficient Public-Key PCF for OT Correlations from Lattices

In this talk



Secret-Power Ring Learning with Errors

Efficient
Pseudorandomly Constrained PRF

CPRF for
(w)PRFs and their complement

PCF for OT

precomputable

PK-PCF for OT
from
Secret-Power RLWE

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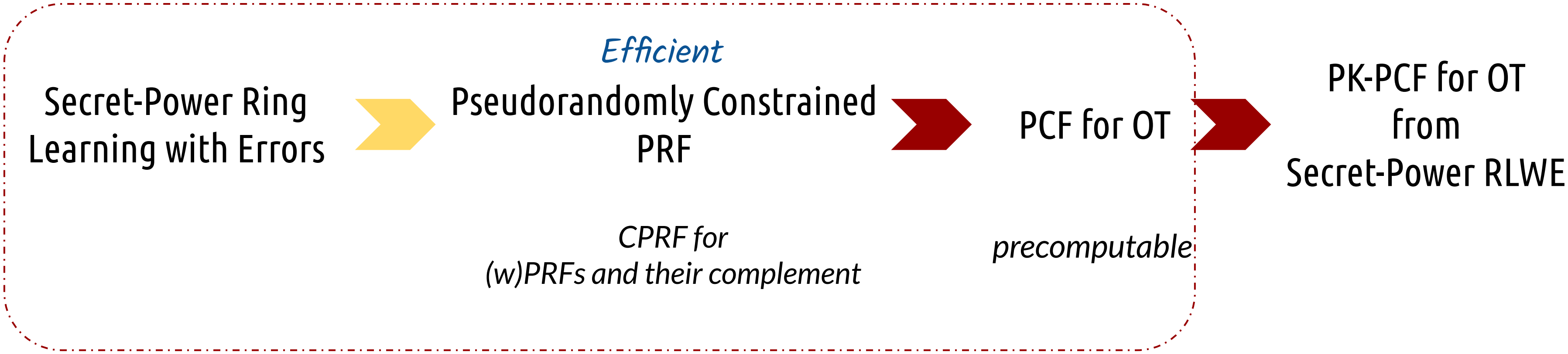

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Efficient

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A PRF
from
Ring LWE

PRFs from Ring LWE

(Ring R_q)

- **Master Secret Key :**

$$\mathsf{msk} := \left((k_1, \ldots, k_n) \xleftarrow{\$} R_q^n,\ s, a \xleftarrow{\$} R_q\right)$$

PRFs from Ring LWE

$P : \{0,1\}^n \times R_q^n \to R_q$

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$(Y \in R_q : \lfloor Y \rceil_2 = \lfloor Y \cdot (2/q) \rceil)$

PRFs from Ring LWE

$P : \{0,1\}^n \times R_q^n \to R_q$ *(with range invertible in R_q)*

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No-Evaluation Security

$as + e$ looks random

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$as + e$ looks random

(Ring LWE)

Full security via Random Oracle

A Constrained PRF
from
Secret-Power Ring LWE

Constrained PRFs from Secret-Power Ring LWE

(Ring R_q)

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Constrained PRFs from Secret-Power Ring LWE

Constraint: ck$_{\vec{z}}$ can evaluate on all $\vec{x}$ iff $P\left(\vec{x}, \vec{z}\right) = 0$ $\left(P : \{0,1\}^n \times R_q^n \to R_q\right)$

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  If $P\left(\vec{x}, \vec{z}\right) = 0$

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is a polynomial in 1/s (rounded mod 2) when P(x,z)=0

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  ➡ $as \cdot P\left(\vec{x}, \vec{k}/s\right) = as \cdot P\left(\vec{x}, \vec{\ell}/s + \vec{z}\right)$

  $= as \cdot P\left(\vec{x}, \vec{z}\right) + a \cdot Q\left(\vec{x}, \vec{z}, 1/s, \vec{\ell}\right)$

  $= \cancel{as \cdot P\left(\vec{x}, \vec{z}\right)} + \sum_{i=0}^{t} \boxed{q_i} \cdot a\left(\frac{1}{s}\right)^i$

  If $P\left(\vec{x}, \vec{z}\right) = 0$

  depends on $\vec{x}, \vec{z}, \vec{\ell}$

Constrained PRFs from Secret-Power Ring LWE

Constraint: $\mathsf{ck}_{\vec{z}}$ can evaluate on all $\vec{x}$ iff $P\left(\vec{x}, \vec{z}\right) = 0$ $\left(P : \{0,1\}^n \times R_q^n \to R_q\right)$

(Ring R_q)

- **Master Secret Key :**

  msk $:= \left((k_1, \ldots, k_n) \xleftarrow{\$} R_q^n, s, a \xleftarrow{\$} R_q\right)$

- **Evaluation on** $\vec{x} = \overbrace{x_1, x_2, \ldots, x_n}^{\text{binary}}$ **:**

  $F_{\mathsf{msk}}\left(\vec{x}\right) := \lfloor as \cdot P\left(\vec{x}, \vec{k}/s\right) \rceil_2$

- **Constrained Key for** $\vec{z} \in \{0,1\}^n$ **:**

  Let $\vec{\ell} = \vec{k} - s \cdot \vec{z}$

Constrained PRFs from Secret-Power Ring LWE

Constraint: ck$_{\vec{z}}$ can evaluate on all $\vec{x}$ iff $P\left(\vec{x},\vec{z}\right)=0$ $\left(P:\{0,1\}^n \times R_q^n \to R_q\right)$

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  Let $\vec{\ell} = \vec{k} - s \cdot \vec{z}$

  $$\text{ck}_{\vec{z}} := \left(\vec{\ell}, \left(\mathcal{A}_i = a \cdot (1/s)^i + e_i\right)_{i\in[0,t-1]}, \vec{z}\right)$$

Constrained PRFs from Secret-Power Ring LWE

Constraint: $\mathsf{ck}_{\vec{z}}$ can evaluate on all $\vec{x}$ iff $P\left(\vec{x}, \vec{z}\right) = 0$ $\left(P : \{0,1\}^n \times R_q^n \to R_q\right)$

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  $$\mathsf{msk} := \left((k_1, \ldots, k_n) \overset{\$}{\leftarrow} R_q^n,\ s, a \overset{\$}{\leftarrow} R_q\right)$$

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- **Constrained Evaluation :**

  1 Find c_i's such that

  $$P\left(\vec{x}, \vec{\ell}/S + \vec{z}\right) = P\left(\vec{x}, \vec{z}\right) + \sum_{i=1}^{t} c_i (1/S)^i$$

Constrained PRFs from Secret-Power Ring LWE

Constraint: ck$_{\vec{z}}$ can evaluate on all $\vec{x}$ iff $P\left(\vec{x}, \vec{z}\right) = 0$ $\left(P : \{0,1\}^n \times R_q^n \to R_q\right)$

(Ring R_q)

- **Master Secret Key :**

  msk $:= \left(\left(k_1, \ldots, k_n\right) \xleftarrow{\$} R_q^n,\ s, a \xleftarrow{\$} R_q\right)$

- **Evaluation on** $\vec{x} = \overbrace{x_1, x_2, \ldots, x_n}^{\text{binary}}$ **:**

  $$F_{\text{msk}}\left(\vec{x}\right) := \lfloor as \cdot P\left(\vec{x}, \vec{k}/s\right) \rceil_2$$

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  $$\text{ck}_{\vec{z}} := \left(\vec{\ell}, \left(\mathcal{A}_i = a \cdot (1/s)^i + e_i\right)_{i \in [0, t-1]}, \vec{z}\right)$$

- **Constrained Evaluation :**

  ① Find c$_i$'s such that

  symbolic variable

  $$P\left(\vec{x}, \vec{\ell}/\boxed{S} + \vec{z}\right) = P\left(\vec{x}, \vec{z}\right) + \sum_{i=1}^{t} c_i (1/\boxed{S})^i$$

Constrained PRFs from Secret-Power Ring LWE

Constraint: ck$_{\vec{z}}$ can evaluate on all $\vec{x}$ iff $P\left(\vec{x}, \vec{z}\right) = 0$ $\left(P : \{0,1\}^n \times R_q^n \to R_q\right)$

(Ring R_q)

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  Let $\vec{\ell} = \vec{k} - s \cdot \vec{z}$

  $$\mathsf{ck}_{\vec{z}} := \left(\vec{\ell}, \left(\mathcal{A}_i = a \cdot (1/s)^i + e_i\right)_{i \in [0, t-1]}, \vec{z}\right)$$

- **Constrained Evaluation :**

  ① Find c$_i$'s such that

  symbolic variable

  $$P\left(\vec{x}, \vec{\ell}/S + \vec{z}\right) = P\left(\vec{x}, \vec{z}\right) + \sum_{i=1}^{t} c_i (1/S)^i$$

  ② Output $\lfloor \sum_{i=0}^{t-1} c_i \cdot \mathcal{A}_i \rceil_2$

Constrained PRFs from Secret-Power Ring LWE

Constraint: $\mathsf{ck}_{\vec{z}}$ can evaluate on all $\vec{x}$ iff $P\left(\vec{x}, \vec{z}\right) = 0$ $\left(P : \{0,1\}^n \times R_q^n \to R_q\right)$

(Ring R_q)

- **Master Secret Key :**

  msk $:= \left((k_1, \ldots, k_n) \overset{\$}{\leftarrow} R_q^n, \; s, a \overset{\$}{\leftarrow} R_q\right)$

- **Evaluation on** $\vec{x} = \overbrace{x_1, x_2, \ldots, x_n}^{\text{binary}}$ **:**

  $F_{\mathsf{msk}}\left(\vec{x}\right) := \lfloor as \cdot P\left(\vec{x}, \vec{k}/s\right) \rceil_2$

Equal when P(x,z)=0

- **Constrained Key for** $\vec{z} \in \{0,1\}^n$

  Let $\vec{\ell} = \vec{k} - s \cdot\cdot \vec{z}$

  $\mathsf{ck}_{\vec{z}} := \left(\vec{\ell}, \left(\mathcal{A}_i = a \cdot (1/s)^i + e_i\right)_{i \in [0, t-1]}, \vec{z}\right)$

- **Constrained Evaluation :**

  ① Find c_i's such that

  symbolic variable

  $$P\left(\vec{x}, \vec{\ell}/S + \vec{z}\right) = P\left(\vec{x}, \vec{z}\right) + \sum_{i=1}^{t} c_i (1/S)^i$$

  ② Output $\lfloor \sum_{i=0}^{t-1} c_i \cdot \mathcal{A}_i \rceil_2$

Constrained PRFs from Secret-Power Ring LWE

Constraint: $\mathsf{ck}_{\vec{z}}$ can evaluate on all $\vec{x}$ iff $P\left(\vec{x}, \vec{z}\right) = 0$ $\left(P : \{0,1\}^n \times R_q^n \to R_q\right)$

(Ring R_q)

- **Master Secret Key :**

  $$\mathsf{msk} := \left((k_1, \ldots, k_n) \xleftarrow{\$} R_q^n, s, a \xleftarrow{\$} R_q\right)$$

- **Evaluation on** $\vec{x} = \overbrace{x_1, x_2, \ldots, x_n}^{\text{binary}}$ **:**

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Equal when P(x,z)=0

& error small

- **Constrained Key for** $\vec{z} \in \{0,1\}^n$ **:**

  Let $\vec{\ell} = \vec{k} - s \cdot \vec{z}$

  $$\mathsf{ck}_{\vec{z}} := \left(\vec{\ell}, \left(\mathcal{A}_i = a \cdot (1/s)^i + e_i\right)_{i \in [0, t-1]}, \vec{z}\right)$$

- **Constrained Evaluation :**

  ① Find c

symbolic variable

Constrained PRFs from Secret-Power Ring LWE

Constraint: ck$_{\vec{z}}$ can evaluate on all $\vec{x}$ iff $P(\vec{x},\vec{z})=0$ $\quad (P:\{0,1\}^n \times R_q^n \to R_q)$

(Ring R_q)

- **Master Secret Key :**

$$\textbf{msk} := \Big((k_1,\overset{\text{small}}{\ldots},k_n) \xleftarrow{\$} R_q^n,\ \overset{\text{small}}{s}, a \xleftarrow{\$} R_q\Big)$$

- **Evaluation on** $\vec{x} = \overbrace{x_1,x_2,\ldots,x_n}^{\text{binary}}$ **:**

$$F_{\text{msk}}(\vec{x}) := \lfloor as \cdot P\left(\vec{x},\vec{k}/s\right)\rceil_2$$

Equal when P(x,z)=0

& error small

- **Constrained Key for** $\vec{z}\in\{0,1\}^n$ **:**

Let $\vec{\ell} = \vec{k} - s\cdot\vec{z}$ small

$$\text{ck}_{\vec{z}} := \left(\vec{\ell}, \left(\mathcal{A}_i = a\cdot(1/s)^i + e_i\right)_{i\in[0,t-1]}, \vec{z}\right)$$

- **Constrained Evaluation :**

① Find c$_i$'s such that

symbolic variable

$$P\left(\vec{x},\vec{\ell}/S+\vec{z}\right) = P(\vec{x},\vec{z}) + \sum_{i=1}^{t} c_i(1/S)^i$$

② Output $\lfloor \sum_{i=0}^{t-1} c_i\cdot\mathcal{A}_i\rceil_2$

Constrained PRFs from Secret-Power Ring LWE

Constraint: ck$_{\vec{z}}$ can evaluate on all $\vec{x}$ iff $P\left(\vec{x}, \vec{z}\right) = 0$ $\left(P : \{0,1\}^n \times R_q^n \to R_q\right)$

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- **Master Secret Key :**

$$\text{msk} := \left((k_1, \dots, k_n) \xleftarrow{\$} R_q^n, \overset{\text{small}}{s}, a \xleftarrow{\$} R_q\right)$$

- **Evaluation on** $\vec{x} = \overbrace{x_1, x_2, \dots, x_n}^{\text{binary}}$ **:**

$$F_{\text{msk}}\left(\vec{x}\right) := \lfloor as \cdot P\left(\vec{x}, \vec{k}/s\right) \rceil_2$$

No-Evaluation Security

$as + e$ looks random given $\left(a \cdot (1/s)^i + e_i\right)_i$

(Secret-Power Ring LWE)

- **Constrained Key for** $\vec{z} \in \{0,1\}^n$ **:**

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Constraint: $\mathsf{ck}_{\vec{z}}$ can evaluate on all $\vec{x}$ iff $P\left(\vec{x}, \vec{z}\right) = 0$ $\left(P : \{0,1\}^n \times R_q^n \to R_q\right)$

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  ② Output $\lfloor \sum_{i=0}^{t-1} c_i \cdot \mathcal{A}_i \rceil_2$

Full security via random oracle

So What?

Our PK-PCF =
Our CPRF from secret-power RLWE
+ Goldreich-Applebaum-Raykov weak PRF ([Gol00,AR16])
+ public-key setup à la succinct HSS ([ARP24])

Work	Post-Quantum	OT Variant	Key-Size	OT/sec
[OSY21]	✗	OT	small	1
[BCMP**R**24]	✗	OT	small	30k
[CDDKS24]	✓	List OT	5.5 MB	1.2M
This Work	✓	OT	567 MB	540-1k
	✓	OT	200 MB	190-450k

All works use random oracles

Summary

Efficient Public-Key PCF for OT Correlations from Lattices

Thank You!

Secret-Power Ring Learning with Errors

Efficient
Pseudorandomly Constrained PRF

CPRF for (w)PRFs and their complement

PCF for OT

precomputable

PK-PCF for OT from Secret-Power RLWE

obtained via succinct HSS techniques

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GAR wPRF
as
a constraint

Goldreich-Applebaum-Raykov weak PRF

[Gol00,AR16]

GAR weak PRF

$\vec{z} \in \{0,1\}^n$, $\vec{x} \in \{0,1\}^{\kappa+\tau}$

$$F\left(\vec{z}, \vec{x}\right) = \mathsf{XOR}_\kappa\text{-}\mathsf{MAJ}_\tau\left(\vec{z}\left[x_1\right], \ldots, \vec{z}\left[x_{\kappa+\tau}\right]\right) \quad \in \{0,1\}$$

where

$$\mathsf{XOR}_\kappa\text{-}\mathsf{MAJ}_\tau\left(b_1, \ldots, b_{\kappa+\tau}\right) = \mathsf{XOR}_\kappa\left(b_1, \ldots, b_\kappa\right) \oplus \mathsf{MAJ}\left(b_{\kappa+1}, \ldots, b_{\kappa+\tau}\right)$$

Goldreich-Applebaum-Raykov weak PRF

[Gol00,AR16]

GAR weak PRF

$\vec{z} \in \{0,1\}^n , \vec{x} \in \{0,1\}^{\kappa+\tau}$

$$F\left(\vec{z},\vec{x}\right) = \mathsf{XOR}_\kappa\text{-}\mathsf{MAJ}_\tau\left(\vec{z}\left[x_1\right],\ldots,\vec{z}\left[x_{\kappa+\tau}\right]\right) \quad \in \{0,1\}$$

where

$$\mathsf{XOR}_\kappa\text{-}\mathsf{MAJ}_\tau\left(b_1,\ldots,b_{\kappa+\tau}\right) = \mathsf{XOR}_\kappa\left(b_1,\ldots,b_\kappa\right) \oplus \mathsf{MAJ}\left(b_{\kappa+1},\ldots,b_{\kappa+\tau}\right)$$

We find two polynomials P and $\overline{P}$ over R_q such that

$$P_{\kappa,\tau}\left(\vec{x},\vec{z}\right) = 0 \iff F\left(\vec{x},\vec{z}\right) = 0$$

$$\overline{P}_{\kappa,\tau}\left(\vec{x},\vec{z}\right) = 0 \iff F\left(\vec{x},\vec{z}\right) = 1$$

→ Degree $O(\kappa+\tau)$